Long-Run Economic Growth

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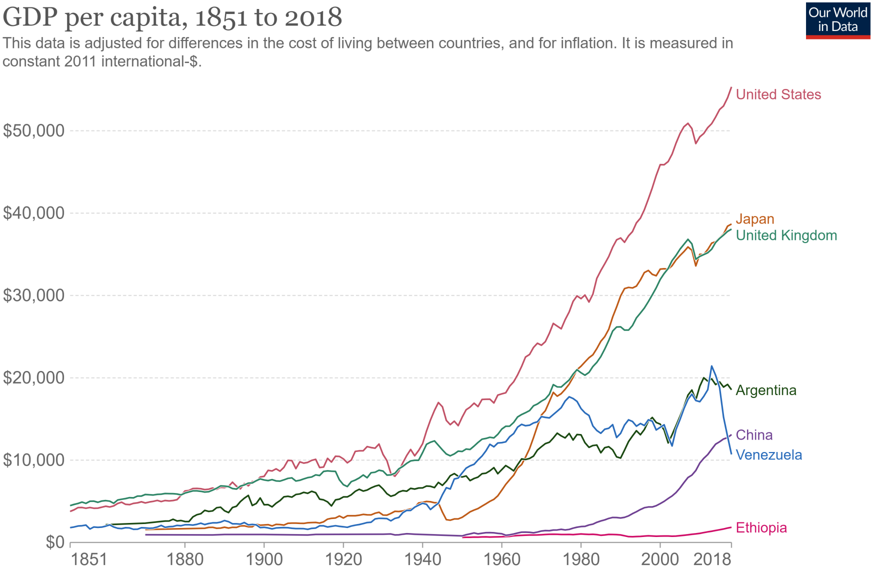
Introduction

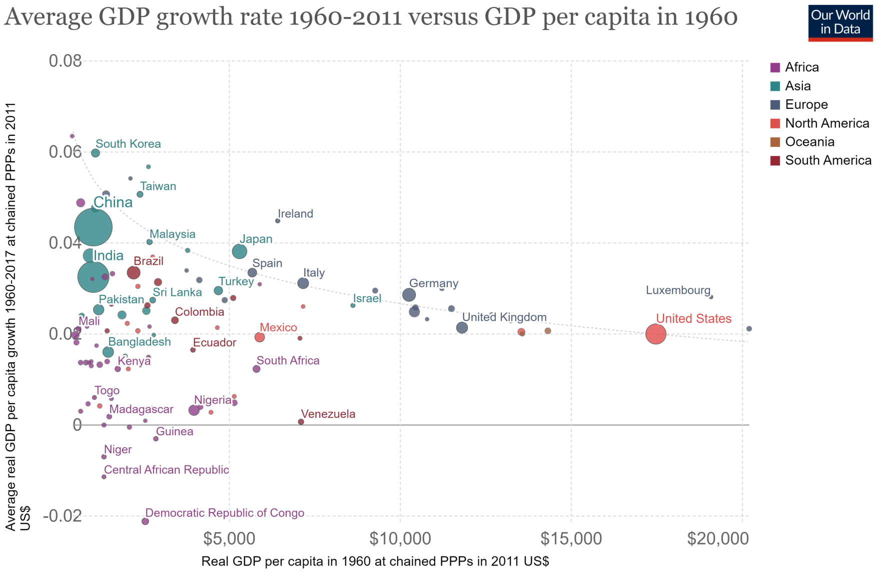
- Macroeconomics studies aggregate economic phenomena in both the medium and short-run (business cycles) and in the long-run (growth)

- The study of economic growth tries to understand why different countries grow at different rates over time

Introduction



Introduction



Introduction

This series of lectures:

1. The sources of economic growth
2. The Solow Model
3. Endogenous Growth Theory
4. Government growth policies

1. The Sources of Economic Growth

Sources of Economic Growth

- The most common measure of growth is (real) GDP growth

$$g_Y = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{\Delta Y_t}{Y_{t-1}}$$

- Recall that the production functions tells us the quantity of goods and services that the economy can produce given its inputs

$$Y_t = A_t F(K_t, N_t)$$

- For an economy to grow, either the **quantities of inputs** or **productivity** must grow

Growth Accounting Equation

- Differentiate the production function with respect to time to obtain the **growth accounting equation**

$$\frac{\Delta Y_t}{Y_{t-1}} = \frac{\Delta A_t}{A_{t-1}} + a_K \frac{\Delta K_t}{K_{t-1}} + a_N \frac{\Delta N_t}{N_{t-1}}$$

- a_K, a_N are the elasticities of output with respect to capital and labor
- They measure the % impact on growth from a 1% increase in each input

$$a_K = \frac{\partial F(K,N)}{\partial K} \frac{K}{F(K,N)}, \quad a_N = \frac{\partial F(K,N)}{\partial N} \frac{N}{F(K,N)}$$

- In the case of the Cobb-Douglas production function:

$$a_K = \alpha$$
$$a_N = 1 - \alpha$$

Growth Accounting Equation

- Decreasing marginal productivity typically implies that $a_K, a_N < 1$
- For the US, $a_K \simeq 0.3$ and $a_N \simeq 0.7$
  - a 1% increase in employment results in 0.7% GDP growth
- Note that productivity has a **one-to-one** impact on growth
- Growth accounting uses the growth accounting equation to measure the relative contributions to growth from the three explicit sources that we consider:
  1. Productivity growth
  2. Growth in capital inputs
  3. Growth in labor inputs

Growth Accounting

How do we implement growth accounting?

1. Measure $\Delta Y_t / Y_{t-1}, \Delta K_t / K_{t-1}, \Delta N_t / N_{t-1}$ directly from the data.
   - Most measures for the growth rate of inputs include quality adjustments, as inputs become more sophisticated over time.
2. Estimate the elasticities a_K, a_N from historical data.
   - This procedure may be complicated due to endogeneity issues, so many economists simply assume $a_K = 0.3, a_N = 0.7$ for the US
3. Calculate the contributions of input growth to economic growth as $a_K \Delta K_t / K_{t-1}$ and $a_N \Delta N_t / N_{t-1}$
4. Obtain the growth of productivity as a **residual** from the GAE

$$\frac{\Delta A_t}{A_{t-1}} = \frac{\Delta Y_t}{Y_{t-1}} - a_K \frac{\Delta K_t}{K_{t-1}} - a_N \frac{\Delta N_t}{N_{t-1}}$$

Growth Accounting Example

Step 1. Obtain measures of output growth, capital growth, and labor growth over the period to be studied.

Example:

$$\text{output growth} = \frac{\Delta Y}{Y} = 40\%;$$
$$\text{capital growth} = \frac{\Delta K}{K} = 20\%;$$
$$\text{labor growth} = \frac{\Delta N}{N} = 30\%.$$

Step 2. Using historical data, obtain estimates of the elasticities of output with respect to capital and labor, a_K and a_N.

Example:

$$a_K = 0.3 \quad \text{and} \quad a_N = 0.7.$$

Step 3. Find the contributions to growth of capital and labor.

Example:

$$\text{contribution to output growth of growth in capital} = a_K \frac{\Delta K}{K} = 0.3 \times 20\% = 6\%;$$
$$\text{contribution to output growth of growth in labor} = a_N \frac{\Delta N}{N} = 0.7 \times 30\% = 21\%.$$

Step 4. Find productivity growth as the residual (the part of output growth not explained by capital or labor).

Example:

$$\text{productivity growth} = \frac{\Delta A}{A} = \frac{\Delta Y}{Y} - a_K \frac{\Delta K}{K} - a_N \frac{\Delta N}{N}$$
$$= 40\% - 6\% - 21\% = 13\%.$$

Growth Accounting: US Application

	(1) 1929–1948	(2) 1948–1973	(3) 1973–1982	(4) 1929–1982	(5) 1982–2017
Source of Growth					
Labor growth	1.42	1.40	1.13	1.34	1.05
Capital growth	0.11	0.77	0.69	0.56	1.17
Total input growth	1.53	2.17	1.82	1.90	2.23
Productivity growth	1.01	1.53	−0.27	1.02	0.97
Total output growth	**2.54**	**3.70**	**1.55**	**2.92**	**3.19**

Sources: Columns (1)–(4) from Edward F. Denison, Trends in American Economic Growth, 1929–1982, Washington, DC: The Brookings Institution, 1985, Table 8.1, p. 111. Column (5) from Bureau of Labor Statistics website, Multifactor Productivity Trends, Table XG, available at `www.bls.gov/mfp/mprdload.htm`.

Productivity Growth in the US

- Contribution of **labor growth** has been slowing due to slowing population growth and aging
- Contribution of **capital growth** has been increasing due to automation and declining price/cost of investment goods
- What about **productivity**?
  1. Very fast growth in the post-war period
  2. Negative growth/slowdown between 1973 and 1982
  3. Rebound after 1982
- Many advanced economies did not benefit as much from rebounding productivity as the US did in recent decades
  - ICT revolution stronger in the US due to govt regulation and competitive pressures

Shortcomings of Growth Accounting

- Pure measurement exercise that takes rates of input growth as given
- Does not explain where input growth comes from, or what policies/institutions can affect those
- As economists, we are interested not just in the measurement, but also on the root causes of growth!
- **Models of economic growth** impose theoretical structure in the study of economic growth, and allow us to investigate the actual sources of growth
- Two types of models:
  1. *Exogenous* growth models study the growth rate of inputs and output, taking the growth of productivity as given
  2. *Endogenous* growth models try to explain where productivity growth comes from
- We begin with the basic framework for studying economic growth, the exogenous growth model developed by Robert Solow in the 1950s

Exogenous vs. Endogenous Variables

- Economists talk about exogenous vs. endogenous variables all the time
- The distinction between exogenous and endogenous variables has to be made in the context of an **economic model**
- In every economic model, some variables are taken as given while others are determined *within the model*, as functions of other model variables and model parameters
- The variables that are taken as given are called **exogenous variables**, they are not "explained" by the model
- Variables that are determined within the model are "explained" by the model and are thus called **endogenous** variables

2. The Solow Model

The Solow Growth Model

The Solow Model allows us to answer two basic questions about growth:

1. What is the relationship between long-run economic growth and variables such as:
   - The saving rate
   - The population growth
   - The rate of technological progress
2. How does economic growth change over time?
   - Does it speed up?
   - Does it slow down?
   - Does it stabilize?

The Solow Growth Model

- Let N_t denote the population in the economy
- For simplicity, we assume that everyone is a worker
- The population grows at rate n:

$$N_{t+1} = N_t(1 + n)$$

- The economy is closed and there are no government purchases, $G = 0$
- Output can be used for consumption or investment

$$Y_t = C_t + I_t$$

- Output is produced using capital and labor

$$Y_t = F(K_t, N_t)$$

- We assume that the production function is homogeneous of degree one: for any constant $\mu > 0$, the production function satisfies

$$F(\mu K_t, \mu N_t) = \mu F(K_t, N_t)$$

Per-Worker Production Function

- It is useful to express all variables in **per capita** (or per worker) ratios

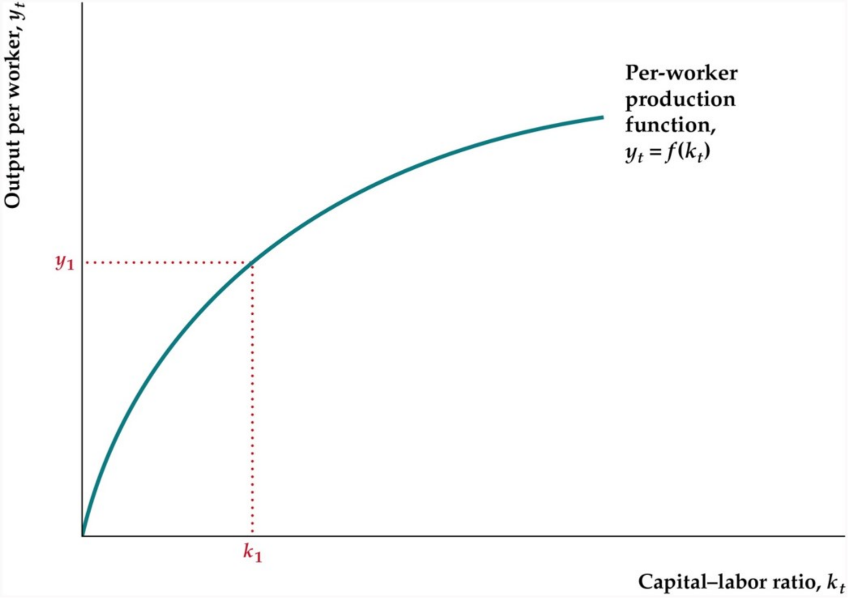
$$y_t = \frac{Y_t}{N_t}, c_t = \frac{C_t}{N_t}, k_t = \frac{K_t}{N_t}$$

- Using the fact that the production function is homogeneous of degree one, we can write the following

$$\begin{aligned} y_t &= \frac{Y_t}{N_t} \\ &= \frac{F(K_t, N_t)}{N_t} \\ &= F\left(\frac{K_t}{N_t}, \frac{N_t}{N_t}\right) \\ &= F(k_t, 1) \equiv f(k_t) \end{aligned}$$

- This is the per-worker production function, and relates output per worker to the capital-labor ratio
- This function has the same shape as the aggregate production function

Per-Worker Production Function



Investment in the Solow Model

- Recall that output can be used for consumption or investment. The **resource constraint** of the economy is
$$Y_t = C_t + I_t$$
- The **law of motion for capital** describes how aggregate capital is accumulated in the economy
$$K_{t+1} = I_t + (1-\delta)K_t$$
- Capital tomorrow is equal to gross investment today plus undepreciated capital
- We can replace the economy's resource constraint in the LoM for capital
$$K_{t+1} = Y_t - C_t + (1-\delta)K_t$$
- Divide through by N_t to write this equation in per worker terms
$$\frac{K_{t+1}}{N_t} = \frac{Y_t}{N_t} - \frac{C_t}{N_t} + (1-\delta)\frac{K_t}{N_t}$$
$$\frac{K_{t+1}}{N_{t+1}}\frac{N_{t+1}}{N_t} = y_t - c_t + (1-\delta)k_t$$
$$k_{t+1}(1+n) = y_t - c_t + (1-\delta)k_t$$

Law of Motion for the Capital-Labor Ratio

- Replace for the per-worker production function $y_t = f(k_t)$ to obtain

$$k_{t+1} = \frac{f(k_t) - c_t + (1-\delta)k_t}{1+n}$$

- This is the **law of motion for the capital-labor ratio**, the fundamental equation of the Solow Model
- It tells us what the capital-labor ratio will be tomorrow, as a function of the capital-labor ratio today, consumption per worker, and model parameters
- Factors that increase the capital-labor ratio tomorrow:
  1. More capital today k_t, as this raises production and investment
  2. Less consumption today c_t, as then a greater share of output is invested
  3. Less population growth n, why? Recall that this is the capital-labor **ratio**: more population growth means that labor is growing faster, which tends to lower this ratio.

Consumption and Saving in the Solow Model

- We assume, for simplicity, that workers save a constant fraction of their income, $s \in [0, 1]$
- This means that consumption per worker is equal to

$$c_t = y_t - s_t = (1 - s)y_t = (1 - s)f(k_t)$$

- Replace in the LoM for k_t:

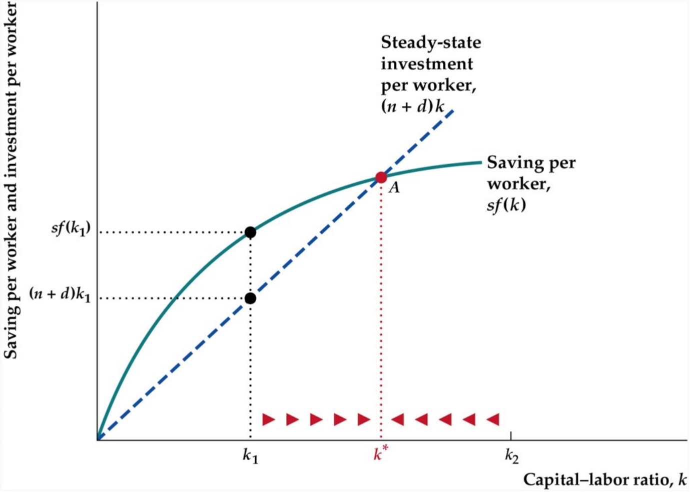
$$k_{t+1} = \frac{sf(k_t) + (1 - \delta)k_t}{1 + n}$$

- Note that a higher savings rate $s \uparrow$ means that workers consume a relatively smaller share of their income and thus save/invest a larger share
- This results in a larger stock of capital tomorrow, $k_{t+1} \uparrow$

Steady State

- A **steady state** is the name given by economists to the equilibrium point of a dynamical system
- In the Solow model, this is a situation where all model variables are constant over time. In particular:
$$k_t = k^*, \forall t$$
- We can solve for the steady state by taking the LoM for k_t and setting
$$k_{t+1} = k_t = k^*$$
- The steady state capital-labor ratio then solves
$$k^* = \frac{sf(k^*) + (1-\delta)k^*}{1+n}$$
- We can simplify this as
$$(n+\delta)k^* = sf(k^*)$$
- Intuitively, this tells us that the steady state is achieved when the rate at which the capital-labor ratio falls (LHS) equals the rate at which the capital-labor ratio grows (RHS)

Steady State



Reaching the Steady State

- Suppose the economy starts with $k_0 < k^*$, i.e. with a level of capital that is below the steady state level
- At this point, the economy will be accumulating capital faster than it depreciates

$$(n+\delta)k_t < sf(k_t)$$

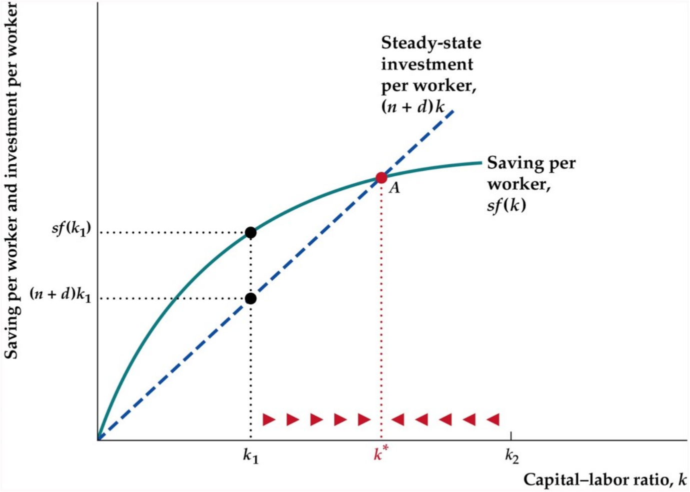
$$\Leftrightarrow k_t < \frac{sf(k_t) + (1-\delta)k_t}{1+n} = k_{t+1}$$

- Over time, capital converges to its steady state level from below

$$k_t \uparrow k^*$$

- Similarly, if the economy starts with $k_0 > k^*$, then it will be depreciating capital faster than it accumulates, thus $k_t \downarrow k^*$

Reaching the Steady State



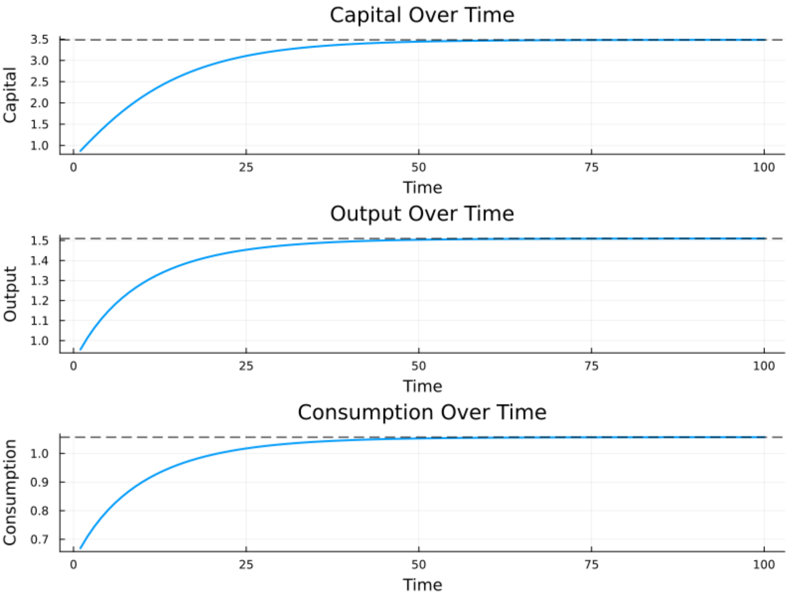
Reaching the Steady State: Numerical Example

- Suppose that the production function is given by $y_t = k_t^\alpha$
- Assume the following parametrization for the model: $\alpha = 0.33, \delta = 0.10, s = 0.30, n = 0.03$
- The steady state capital-labor ratio is then

$$k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{0.3}{0.1+0.03}\right)^{\frac{1}{1-0.33}} = 3.48$$

- Let's assume that the economy starts with $k_0 = 0.25 \times k^*$ and use the LoM to simulate the economy's transition to the steady state
- Very easy to do in Python, Julia, Matlab, etc. (even in Excel!)

Reaching the Steady State: Numerical Example



Long-Run Growth and Steady States

- Once the economy reaches its steady state, the capital-labor ratio is constant
- This means that output and consumption per worker will also be constant

$$
\begin{aligned}
y^* &= f(k^*) \\
c^* &= (1-s)f(k^*)
\end{aligned}
$$

- Note, however, that since we assume population growth, aggregate capital, output and consumption will be growing at rate n

$$
K_t^* = k^* \times N_t \Rightarrow \frac{\Delta K_t^*}{K_{t-1}^*} = n
$$

- At the steady state, all growth is driven by the exogenous population growth rate
- That is why this is called an **exogenous growth model**
- Easy to add productivity growth $\Delta A_t / A_{t-1} = g$ (homework!)

Factors that affect the steady state

- The steady state capital-labor ratio is the solution to an implicit equation that depends on the function f and the parameters (s, δ, n)

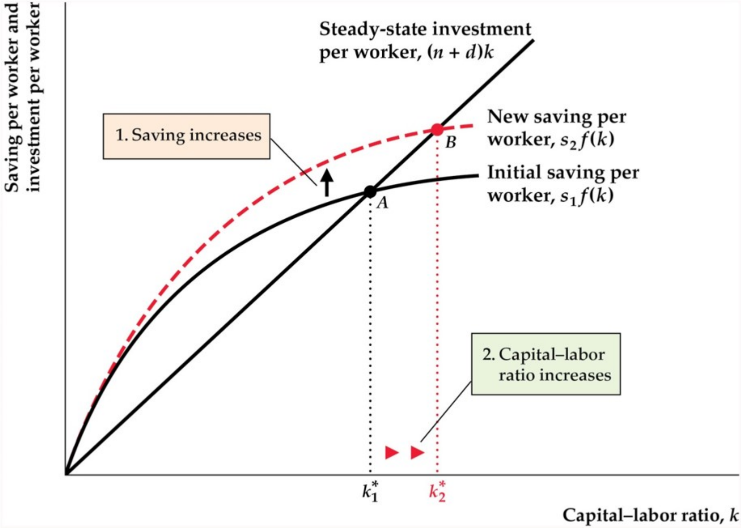
$$(n + \delta)k^* = sf(k^*)$$

- Example: assume that $y_t = ak_t^{\alpha}$, then we can solve for steady state capital in closed form:

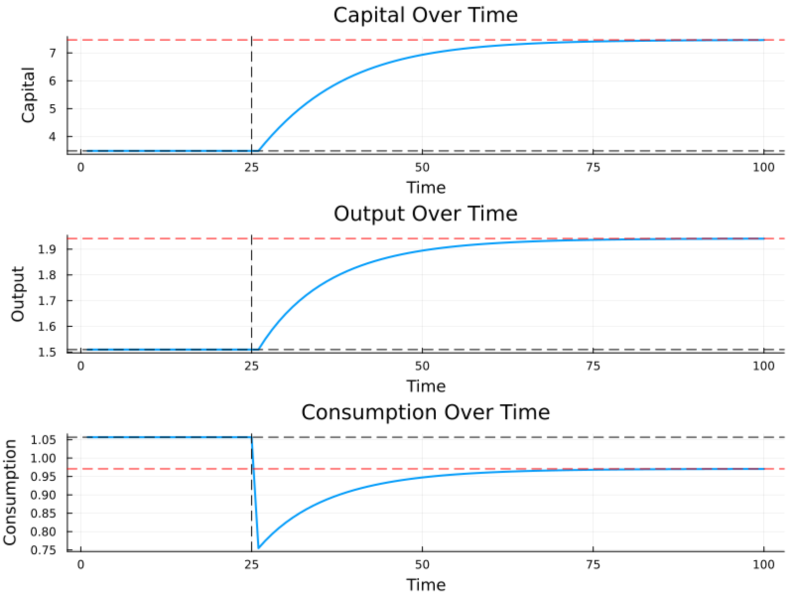
$$k^* = \left(\frac{sa}{n + \delta}\right)^{\frac{1}{1-\alpha}}$$

- Steady state capital increases with: (i) an increase in the savings rate $s \uparrow$, (ii) a decrease in the population growth rate $n \downarrow$, (iii) an increase in productivity $a \uparrow$

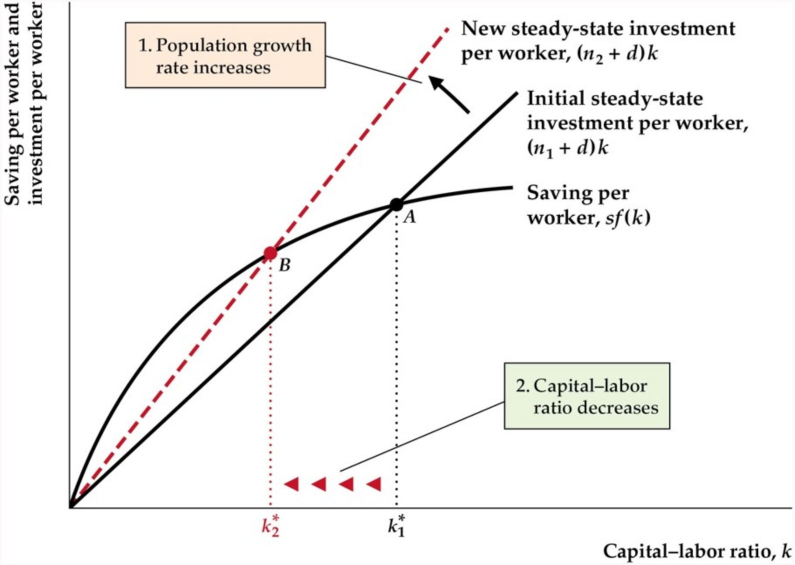
Increase in the Saving Rate



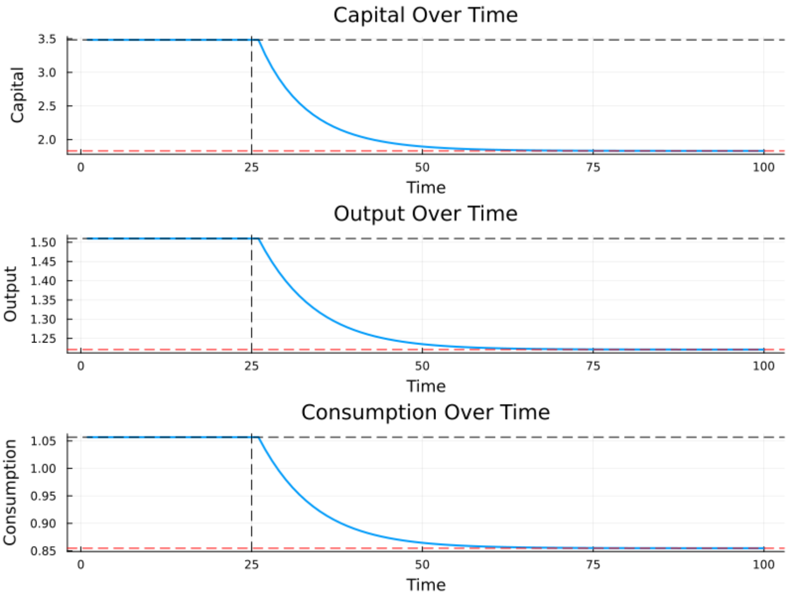
Increase in the Saving Rate: Numerical Example



Increase in Population Growth



Increase in Population Growth: Numerical Example



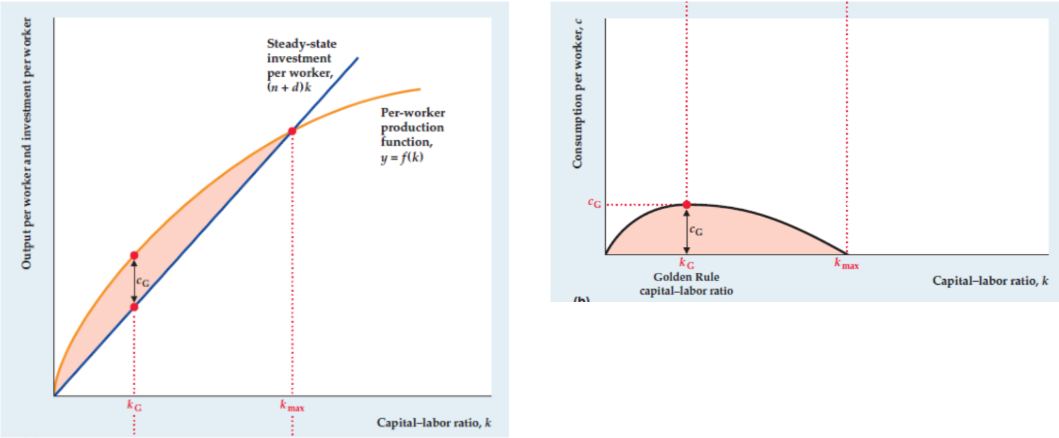
Policy in the Solow Model

- Changes in s or n shift an economy's SS, but do not change long-run growth (as the economy will eventually settle in a new SS anyway)
- Trying to reduce n can lead to other long-run issues that ultimately reduce growth (ex: European and East Asian countries)
- What about the savings rate? At the end of the day, people care about *consumption*, not capital or output

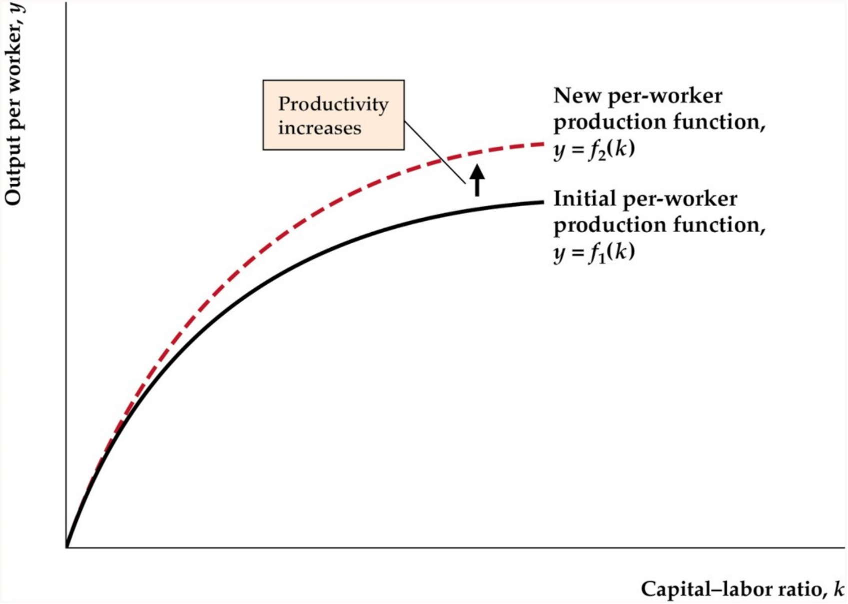
$$c^* = (1 - s)f(k^*)$$

- Increasing the savings rate has an ambiguous effect on consumption:
  1. on one hand, it directly lowers consumption today
  2. however, it raises the SS capital-labor ratio $k^* \uparrow$, which may offset this effect
- The savings rate that maximizes steady state consumption is called the **Golden Rule savings rate**, s^{GR}

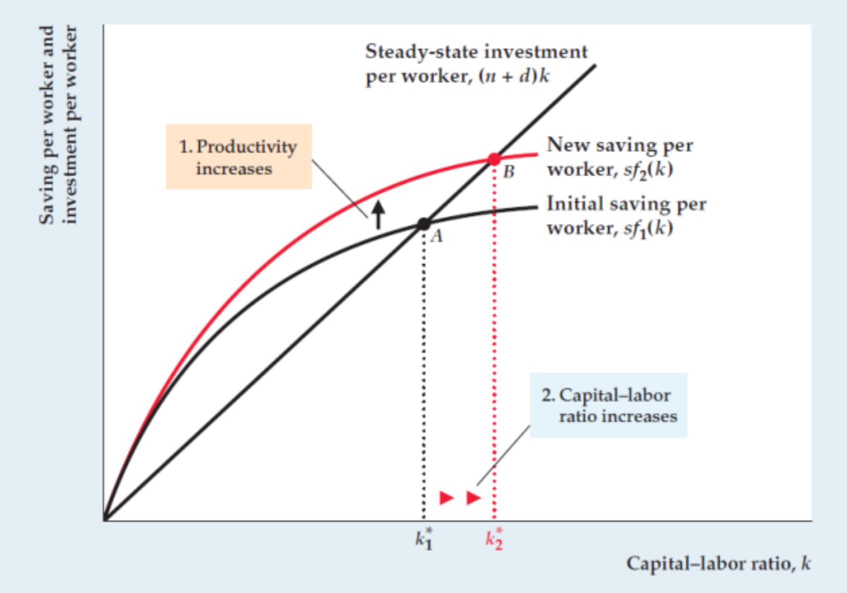
Golden Rule Savings Rate



Productivity Growth



Productivity Growth



Productivity Growth

- Ultimately, productivity A_t is the key factor in economic growth
- Easy to extend the model to allow for this, assume that $\Delta A_t / A_{t-1} = g$ and that the production function is now

$$Y_t = A_t F(K_t, N_t)$$

- Instead of per worker, define variables *per efficiency unit of labor*

$$\tilde{k}_t = \frac{K_t}{A_t N_t}$$
$$\tilde{y}_t = \frac{Y_t}{A_t N_t} = f(\tilde{k}_t)$$
$$\tilde{c}_t = \frac{C_t}{A_t N_t}$$

Productivity Growth

- The LoM for the capital-labor ratio is now given by

$$\tilde{k}_{t+1}(1+n)(1+g) = sf(\tilde{k}_t) + (1-\delta)\tilde{k}_t$$

- The steady state is implicitly defined as

$$\tilde{k}^*(n+g+ng+\delta) = sf(\tilde{k}_t)$$

- At the SS, aggregate variables grow at a rate $(1+n)(1+g)$

- Per capita variables grow at rate g

3. Endogenous Growth Theory

From exogenous to endogenous growth

- In the Solow Model, the only determinant of long-run growth in per capita incomes is productivity growth, which is **exogenous**
- That is, the Solow Model assumes that productivity grows, instead of trying to explain why it grows
- This motivated some economists in the 1980s, such as Paul Romer, to try to develop a theory of why productivity grows
- This class of theories is called **endogenous growth theory**

AK production

- Assume that the population/number of workers is constant
- Instead of a more traditional decreasing marginal returns production function, let us assume that output is linear in capital

$$Y = AK$$

- Note that $MPK = A$ is constant!
- Different theories of endogenous growth try to provide a justification for why this is the case

Why is the MPK constant?

Some of the most influential explanations include:

1. **Human Capital**: as economies become richer, they devote more resources to education and investment in people's skills. This increases the economy's "human capital", which in turn increases productivity. This increase in productivity offsets the diminishing marginal returns on physical capital.

2. **R&D**: As the economy grows, firms have incentives to invest in research and development, which leads to increases in productivity that offset diminishing marginal returns of capital.

Endogenous Growth Model

- Assume again that savings (and consumption) are a constant fraction of output

$$S_t = sAK_t$$

- Recall that investment equals net investment plus depreciation

$$I_t = K_{t+1} - (1-\delta)K_t$$

- Equilibrium in the goods market means that investment is equal to savings, thus

$$sAK_t = K_{t+1} - (1-\delta)K_t$$

- We can then write the growth rate of capital as

$$\begin{aligned} \frac{\Delta K_{t+1}}{K_t} &= \frac{K_{t+1} - K_t}{K_t} \\ &= \frac{sAK_t + (1-\delta)K_t - K_t}{K_t} \\ &= sA - \delta \end{aligned}$$

Endogenous Growth Model

- Since output is proportional to capital, the two growth rates must coincide

$$\frac{\Delta Y_{t+1}}{Y_t} = \frac{\Delta K_{t+1}}{K_t}$$

- Thus the growth rate of output is also equal to $sA - \delta$
- Note that long-run growth now depends on the savings rate s, unlike in the Solow growth model
- The idea is that higher rates of saving lead to more investment in MPK-offsetting technologies such as education and R&D
- Endogenous growth theory shows that variables such as the saving rate, which can be directly affected by government policies, contribute to the economy's long-run growth

4. Government Growth Policies

Government Growth Policies

- Policies to increase growth are crucial political issues both in advanced and developing economies
- The Solow Model and the EGM show us that two key variables that affect long-run economic growth are
  - Productivity growth
  - The saving rate

Policies to Affect the Saving Rate

- If private markets are efficient, there is little rationale for the government to interfere with private decisions to save and consume
- As we saw in previous lectures, private savings arise from an optimal decision to trade off current and future consumption
- Some people believe, however, that behavioral and tax distortions generate saving rates that are too low in countries such as the US
- Fixing tax distortions, for example, would effectively raise the real return on savings. But empirical evidence shows that the response of aggregate savings to the real rate is relatively small
- Another alternative is to increase government saving, by reducing deficits and paying off the national debt
  - Ricardian Equivalence implies that nothing will happen unless the government changes its spending policies!

Policies to Raise Productivity Growth

- The scope to increase savings rates seems to be relatively limited in free-market advanced economies such as the US
- More attention tends to be paid to policies that promote long-run productivity growth
    1. Infrastructure investment: there is empirical evidence that quality of infrastructure affects productivity, and US infrastructure spending has declined in the last decades
    2. Building human capital: there is a very strong empirical link between productivity and human capital, and the government can encourage human capital formation through education and work training investment
    3. Research and Development: the government can support R&D by supporting and funding scientific research (i.e, NSF), government research facilities (i.e., NASA), contracts for particular projects, etc.