Productivity, Output, and Employment

Econ 4021, Washington University in St. Louis

Miguel Faria-e-Castro
Federal Reserve Bank of St. Louis

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Introduction

- Previous lectures: how to measure economic variables.
- Now, we turn to understanding how these variables are determined
- Key variables like output, inflation, and unemployment are jointly determined in a system of markets where different agents interact:
  - Firms
  - Households
  - Government
- To begin, we analyze what drives the behavior and decisions of these agents in various markets.
- Today, we focus specifically on the **labor market**.

Introduction

This series of lectures:

1. The Production Function
2. The demand for labor
3. The supply of labor
4. Labor market equilibrium
5. Unemployment
6. Output and unemployment: Okun's Law

1. The Production Function

The Production Function

- The production function describes how the economy transforms inputs into output:

$$Y = AF(K, N)$$

- Key components:
  - K: Capital (e.g., machinery, buildings, tools)
  - N: Labor (e.g., hours worked, workforce size)
  - These are the primary **factors of production**.
  - Additional factors (e.g., energy, raw materials) may also play a role in practice.
- F: A function that translates input quantities (K, N) into output (Y).
- A: **Total Factor Productivity (TFP)**, a measure of the efficiency of input use. Could reflect (among others):
  - Quality of management
  - Natural conditions
  - Available technology

The US Production Function

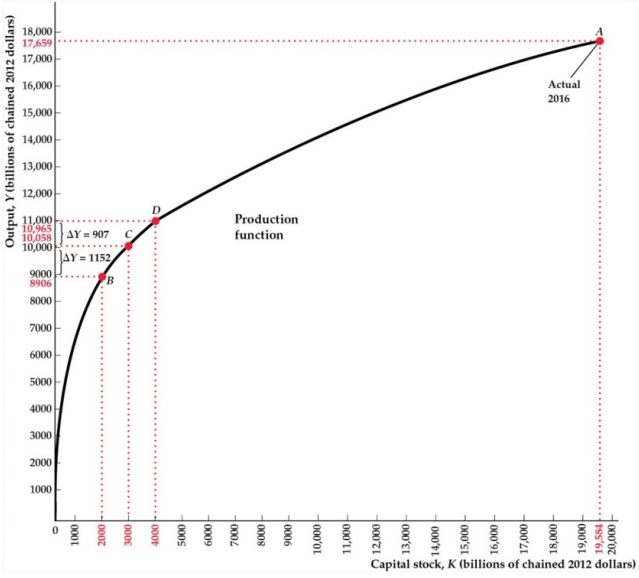
- A common type of production function is the **Cobb-Douglas production function**, which takes the form

$$Y = AK^{\alpha}N^{1-\alpha}$$

- The way that the US economy converts inputs to output is reasonably well approximated by a Cobb-Douglas production function with parameter $\alpha = 0.3$

$$Y = AK^{0.3}N^{0.7}$$

The Production Function



Marginal Product

- The **marginal product** measures the change in output from a one-unit increase in an input, holding other inputs constant.
- Marginal products for capital and labor:

$$MPK = \frac{\partial Y}{\partial K} = A\alpha K^{\alpha-1}N^{1-\alpha}, \quad MPN = \frac{\partial Y}{\partial N} = AK^{\alpha}(1-\alpha)N^{-\alpha}$$

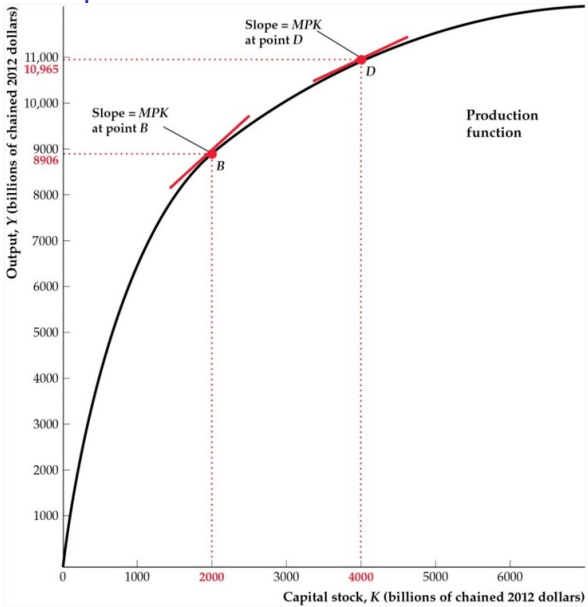
- Typical assumptions about the production function:
  1. **Positive marginal products**: Increasing capital or labor increases output.

$$\frac{\partial Y}{\partial K} > 0, \quad \frac{\partial Y}{\partial N} > 0$$

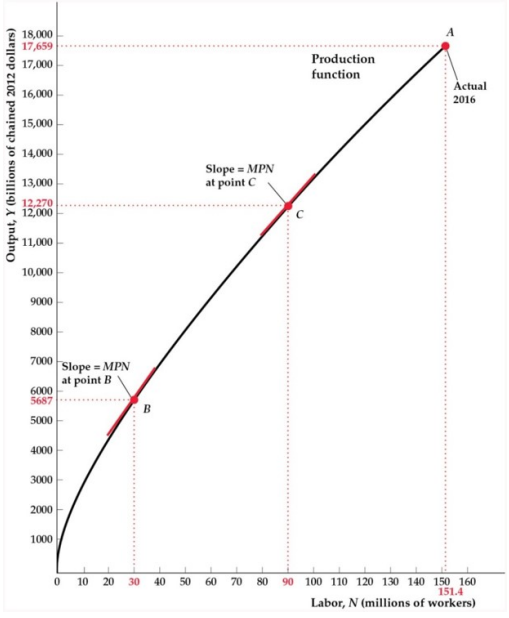
  2. **Diminishing marginal products**: The additional output from an extra unit of input decreases as the input quantity rises.

$$\frac{\partial^2 Y}{\partial K^2} < 0, \quad \frac{\partial^2 Y}{\partial N^2} < 0$$

Marginal Product of Capital



Marginal Product of Labor

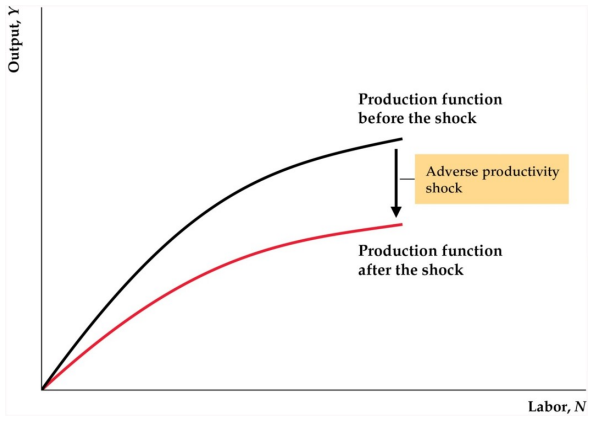


Productivity Shocks

- **Productivity shocks** are a type of supply shock that cause changes in the economy's production function.
- A productivity shock occurs when **total factor productivity** (A) changes, leading to variations in output for the same quantity of inputs.
- These shocks can be:
  - **Positive:**
    - Technological progress increases A, allowing the economy to produce more output with the same inputs.
  - **Negative:**
    - Natural disasters or disruptions reduce A, limiting the ability to combine inputs effectively to produce output.
- Productivity shocks play a central role in prominent theories of the business cycle (i.e. Real Business Cycle theory)

Productivity Shocks: movements of vs. along the curve

- Productivity shocks are shifts of the production function
- Contrast with changes in the quantity of inputs which are movements along the production function



2. The Demand for Labor

Demand for Labor

- Firm inputs: capital K and labor N
- Capital is relatively slow to adjust: firms need to invest in new capital, or scrap/sell old capital, and this takes time
- Labor can be adjusted relatively quickly: fire workers, ask workers to work overtime, etc.
- In the **short-run** we treat capital as fixed and labor as flexible

Simplifying assumptions:

1. Workers are all alike (no differences in skill, willingness to work, etc.)
2. Labor markets are competitive and firms take the prevailing wage as given
3. Firms maximize profits

Problem of the Firm

$$\max_N P \times AF(K, N) - W \times N - R \times K$$

- P, W, R are the nominal price of output, wage, and user cost of capital
- Capital is fixed: K is taken as given
- Competitive labor markets: nominal wage W is taken as given
- First-order condition with respect to labor:

$$\underbrace{P \times A \frac{\partial F(K, N)}{\partial N}}_{\text{marginal revenue of labor} = P \times MPN} = W$$

- Marginal revenue of labor is equated to nominal wage
- Alternatively, we can say that the MPN is equal to the **real wage** $w = \frac{W}{P}$
- Firms hire workers until the revenue of hiring an extra worker is equal to its cost

Analysis at the Margin

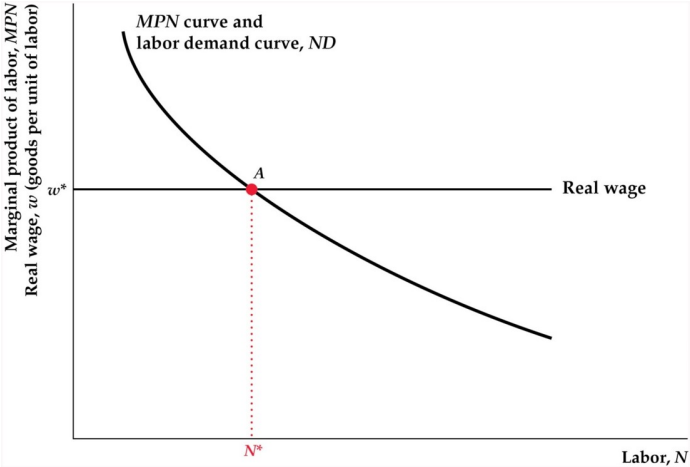
- Recall that we assumed **diminishing marginal productivity of labor**

$$\frac{\partial^2 Y}{\partial N^2} < 0$$

- Second-derivative wrt labor is negative $\Rightarrow$ first derivative wrt labor is **decreasing**
- In other words, MPN is **decreasing** in the quantity of labor
- If $MPN > w$, hiring an extra worker produces more revenue than it costs, so $N \uparrow$
- If $MPN < w$, then the firm could fire a worker and increase its profits, so $N \downarrow$
- Firm profits are maximized over N when $MPN = w$

Labor demand

The MPN curve is the labor demand curve for the firm



The optimal amount of labor N^* is the point where the labor demand curve intercepts the real wage

Example: Cobb-Douglas Production Function

Recall that the Cobb-Douglas production function is given by

$$Y = AK^{\alpha}N^{1-\alpha}$$

The firm's problem is

$$\max_{N} AK^{\alpha}N^{1-\alpha} - wN - rK$$

The firm's optimality condition is then

$$\frac{\partial AK^{\alpha}N^{1-\alpha}}{\partial N} = AK^{\alpha}(1-\alpha)N^{-\alpha} = w$$

We can then solve for the optimal number of workers the firm would like to hire given wages, capital, and productivity

$$N^{*} = \left[\frac{AK^{\alpha}(1-\alpha)}{w}\right]^{\frac{1}{\alpha}}$$

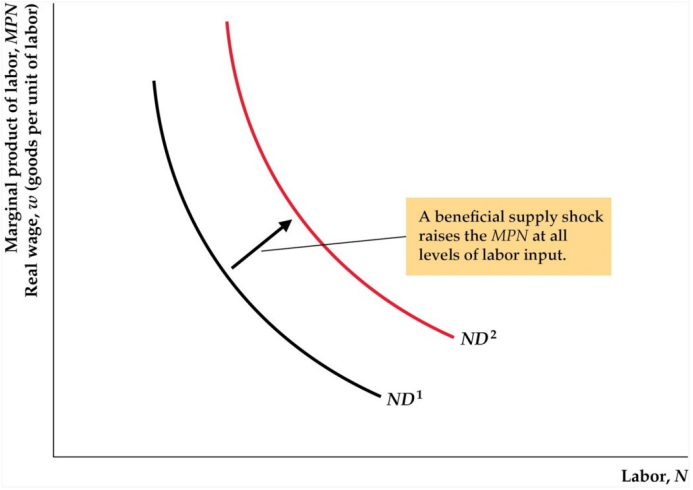
Shifts in Labor Demand

- A change in the wage causes a movement **along** the labor demand curve, not a shift of the curve
- A change in any other variable that affects the MPN **shifts** labor demand
- Recall the Cobb-Douglas example:

$$MPN = AK^{\alpha}(1-\alpha)N^{-\alpha}$$

- Changes in TFP A or in capital K shift the labor demand curve
- Increases in A or K are **positive supply shocks** that shift curve to the right
  - Firms willing to hire more workers at the same real wage w

Shifts in Labor Demand



Aggregate Labor Demand

- In macroeconomics, we typically work with the **aggregate demand for labor**
- This is the sum of the labor demand of all individual firms
- In an economy with J firms, aggregate labor demand is given by

$$N^d(w) = \sum_{j=1}^{J} N_j^d(w)$$

- In the Cobb-Douglas case:

$$N^d(w) = \sum_{j=1}^{J} \left[\frac{A_j K_j^{\alpha}(1-\alpha)}{w} \right]^{\frac{1}{\alpha}}$$

- If all firms have the same productivity and capital:

$$N^d(w) = J \times \left[\frac{A K^{\alpha}(1-\alpha)}{w} \right]^{\frac{1}{\alpha}}$$

3. The Supply of Labor

The Supply of Labor

- The demand for labor is determined by firms, while the **supply of labor** is determined by workers or unions
- Households observe the equilibrium wage (w) and decide:
  - How much labor to supply to firms.
  - How much time to allocate to other activities.
- The cost of working is the **opportunity cost** of time, the value of alternative uses:
  - Leisure
  - Home production
  - Education
- The **aggregate supply of labor** represents the total labor supplied by all individuals in the economy

The Income-Leisure Trade-off

- Economists use the concept of **utility** to measure individuals' happiness or satisfaction
- Utility is assumed to increase with the consumption of goods C and services as well as with leisure time L:

$$U = u(C, L)$$

  where u is a **utility function** that is increasing in C and L
- Utility levels are irrelevant - what matters is how the utility of different bundles (C, L) compare through the function u

$$u(C_1, L_1) > u(C_2, L_2) \quad \Rightarrow \text{Bundle } (C_1, L_1) \text{ is preferred to } (C_2, L_2).$$

- Common assumptions:
  1. **More is better:** marginal utility is positive.
  2. **Diversity is desirable:** preferences are convex.
  3. **Normal goods:** desired quantities of C and L increase with income.

The Income-Leisure Trade-off

- Households want as much consumption and leisure as possible
- However, individuals need income in order to consume, and that income must be earned by working
- Individuals have limited time in a day, and so working takes time away from leisure
- This trade-off can be represented with two mathematical constraints
- First, the **time constraint** states that the individual has limited time that must be allocated between work N and leisure L

$$N + L = 24$$

The Income-Leisure Trade-off

- Second, the **budget constraint** states that consumption expenditure must not exceed income, which is earned by working

$$C = wN$$

where w is the real wage per hour worked

- Recall that $N = 24 - L$
- We can combine the two constraints into a single budget constraint that illustrates the income-leisure trade-off:

$$C = w(24 - L)$$

- If you want more consumption, you need to forego leisure, and vice-versa

The Income-Leisure Trade-off

- We assume that the marginal utilities of consumption and leisure are positive

$$\frac{\partial u(C,L)}{\partial C} > 0, \frac{\partial u(C,L)}{\partial L} > 0$$

this means that individuals always like more consumption and more leisure

- We also assume **decreasing marginal utilities** of consumption and leisure

$$\frac{\partial^2 u(C,L)}{\partial C^2} < 0, \frac{\partial^2 u(C,L)}{\partial L^2} < 0$$

- Each additional unit of consumption and/or leisure generates a decreasing amount of utility
- This is the law of diminishing returns applied to the household problem

The Income-Leisure Trade-off

- The household's decision problem can be expressed as

$$\max_{C,L} u(C, L)$$
$$\text{subject to } C = w(24 - L)$$

- Replace the constraint in the utility function to obtain

$$\max_{L} u[w(24 - L), L]$$

- Univariate problem $\Rightarrow$ easy to solve! First-order condition:

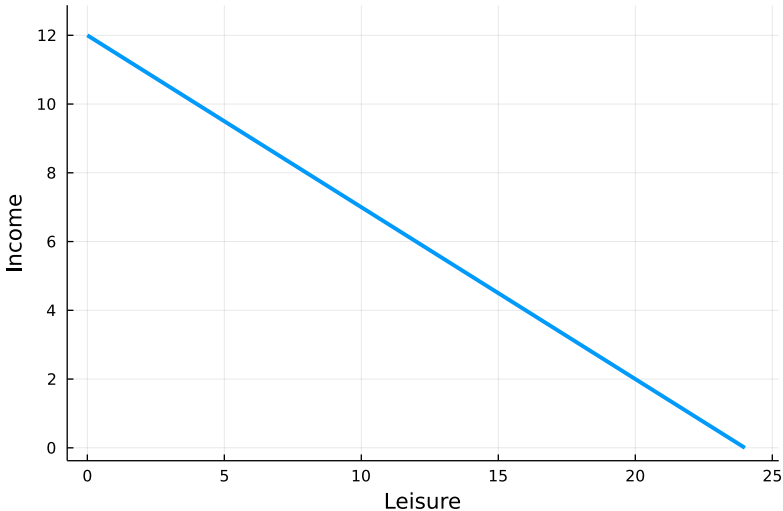
$$\frac{\mathrm{d}u(C, L)}{\mathrm{d}L} = \frac{\partial u(C, L)}{\partial C}(-w) + \frac{\partial u(C, L)}{\partial L} = 0$$

- Familiar condition: MRS equals relative prices

$$MRS_{C,L} = \frac{\frac{\partial u(C,L)}{\partial L}}{\frac{\partial u(C,L)}{\partial C}} = w$$

The Income-Leisure Trade-off

The budget constraint $C = w(24 - L)$ for $w = 0.5$



The Income-Leisure Trade-off

- The wage rate is the *relative price of leisure in terms of consumption*
- By enjoying an extra hour of leisure, the household foregoes w units of real consumption
- Households chooses a combination of consumption and leisure such that the ratio of marginal utilities equals the wage
- This takes into account the consumption benefit of working an extra hour (consume an extra w units) and the leisure cost of working that extra hour

Example

Consider the following utility function

$$u(C, L) = \sqrt{C} + \eta\sqrt{L}$$

The household problem is then

$$\max_{L} \sqrt{w(24 - L)} + \eta\sqrt{L}$$

Taking the first-order conditions:

$$\sqrt{w}\frac{1}{2}\frac{1}{\sqrt{24 - L}}(-1) + \eta\frac{1}{2}\frac{1}{\sqrt{L}} = 0$$

We can solve for optimal leisure as

$$L^* = \frac{24\eta^2}{w + \eta^2}$$

$$\Rightarrow N^* = 24 - L = 24\left[1 - \frac{\eta^2}{w + \eta^2}\right]$$

Example

Optimal leisure and labor are

$$L^* = \frac{24\eta^2}{w + \eta^2}$$

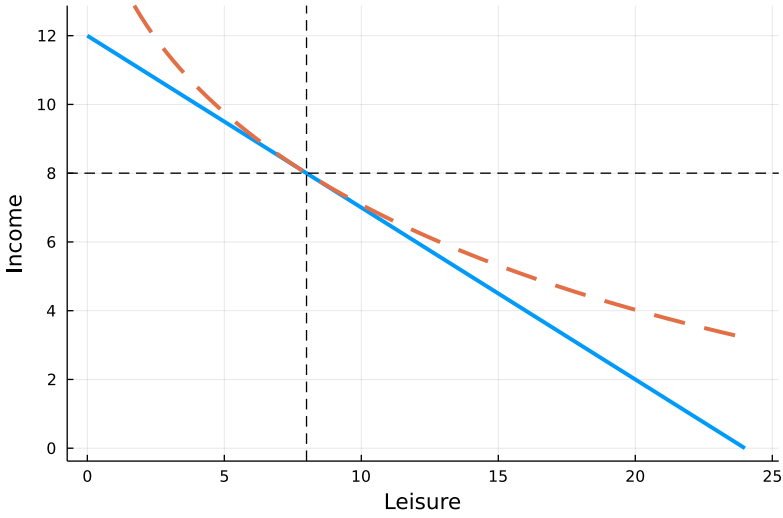
$$N^* = 24\left[1 - \frac{\eta^2}{w + \eta^2}\right]$$

We can then use the budget constraint to solve for optimal consumption

$$\begin{aligned} C^* &= w \times N^* \\ &= w \times 24\left[1 - \frac{\eta^2}{w + \eta^2}\right] \end{aligned}$$

Example

Optimal bundle is where the budget constraint is tangent to the indifference curve



Real Wages and Labor Supply

- The **real wage** represents the purchasing power of the payment for an hour of work:

$$w = \frac{W}{P}$$

- What happens when the real wage increases?

  1. The benefit of working an additional hour increases, which induces people to supply more labor. This is the **substitution effect**.
  2. But, for the same amount of hours worked, workers earn a higher income and could thus afford the same amount of consumption by working less hours and enjoying more leisure. This induces people to supply less labor and is called the **income effect**.

- These two forces have opposite effects on labor supply

Substitution Effect

- If $w \uparrow$, working an extra hour leads to more resources available for spending on consumption
- This induces people to work more, everything else constant
- Temporary increases in wages are likely to generate "pure substitution effects"

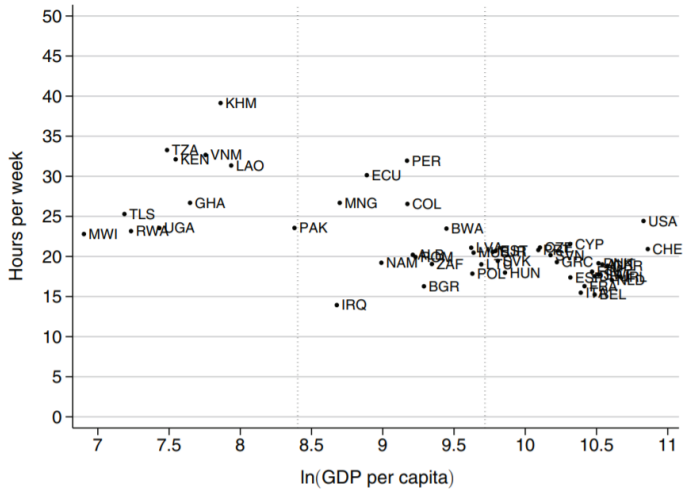
Income Effect

- Any increase in income (related or unrelated to labor) means that a worker can afford the same level of consumption while working fewer hours
- It could be unrelated to labor: inheritance, lottery wins, government transfers
- This is called the income effect or, sometimes, depending on the context, the wealth effect
- It could also be related to labor: a future increase in the wage, for example

Combining the Two Effects

- An increase in the real wage increases hours worked via the substitution effect, and decreases hours worked via the income effect
- The net effect is in principle uncertain and depends on several factors, such as:
  1. The magnitude of the increase in the wage
  2. The elasticity of labor supply
- Empirical evidence: substitution effects tend to dominate for temporary changes in real wages, while income effects dominate for permanent changes

Hours worked vs. GDP per capita



Source: Bick et al., 2018 "How do Hours worked vary with income? Cross-country evidence and implications", American Economic Review

Labor Supply

Amount of labor N that a worker supplies at a given real wage w

The optimal amount of labor is the point where the labor supply curve intercepts the prevailing real wage.

Example

Recall our utility function example

$$N^s = 24\left[1 - \frac{\eta^2}{w + \eta^2}\right]$$

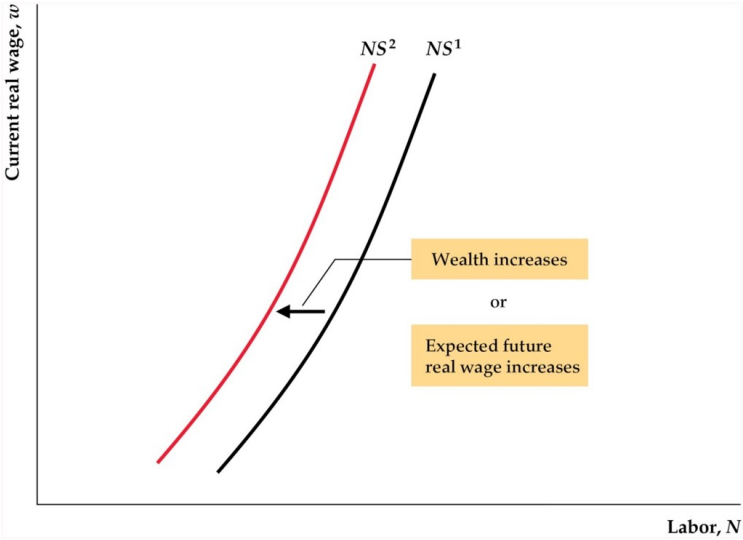
Note that labor supply is *increasing* in the real wage.

- What does this mean regarding income vs. substitution effects?

Shifts in Labor Supply

- A change in the wage causes a movement **along** the labor supply curve, not a shift of the curve
- Changes in any other variables that affect the leisure-consumption trade-off lead to a shift in the curve
- For ex: non-wage related changes in wealth trigger income effects that shift the labor supply curve
- An increase in wealth means that the worker supplies less hours for the same wage $\Rightarrow$ labor supply shifts to the left

Shifts in Labor Supply



Aggregate Labor Supply

- The **aggregate labor supply** is the total amount of labor supplied by all workers in an economy for a given real wage
- In an economy with I workers, aggregate labor supply is given by

$$N^s(w) = \sum_{i=1}^{I} N_i^s(w)$$

- In our previous example:

$$N^s(w) = \sum_{i=1}^{I} 24 \left[1 - \frac{\eta_i^2}{w + \eta_i^2}\right]$$

- If all workers have the same $\eta_i = \eta, \forall i$:

$$N^s(w) = I \times 24 \left[1 - \frac{\eta^2}{w + \eta^2}\right]$$

Aggregate Labor Supply

Economy-wide increases in the real wage tend to increase the aggregate quantity of labor that is supplied in two ways:

1. For people who are currently working, the substitution effect may dominate and they may supply more hours. This is called the **intensive margin**.

2. Some people may have dropped out of the labor force but may be enticed to return if wages are high enough. This is called the **extensive margin**.

People who are out of the labor force include:

- Retirees
- Adults who attend school full time
- Adults who engage in domestic production full time

Shifts in Aggregate Labor Supply

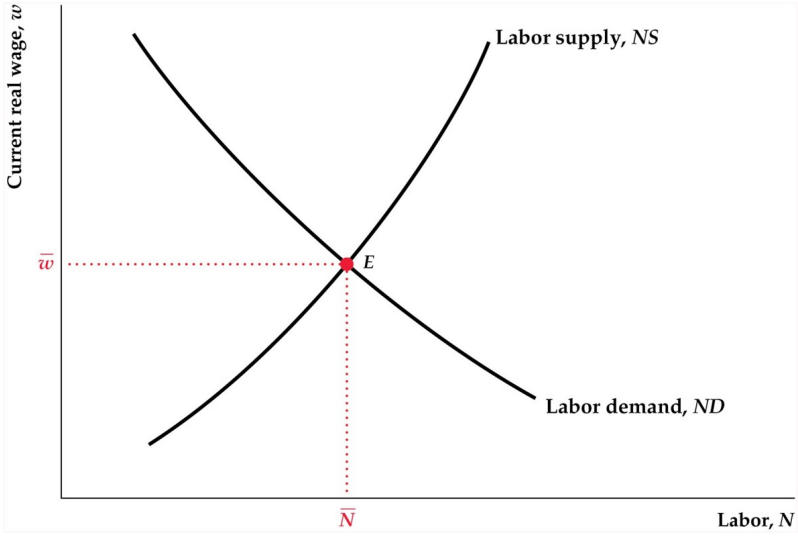
Factors That Shift the Aggregate Labor Supply Curve

An increase in	Causes the labor supply curve to shift	Reason
Wealth	Left	Increase in wealth increases amount of leisure workers can afford.
Expected future real wage	Left	Increase in expected future real wage increases amount of leisure workers can afford.
Working-age population	Right	Increased number of potential workers increases amount of labor supplied.
Participation rate	Right	Increased number of people wanting to work increases amount of labor supplied.

4. Labor Market Equilibrium

(Classical) Labor Market Equilibrium

Combine supply and demand to obtain the labor market equilibrium:

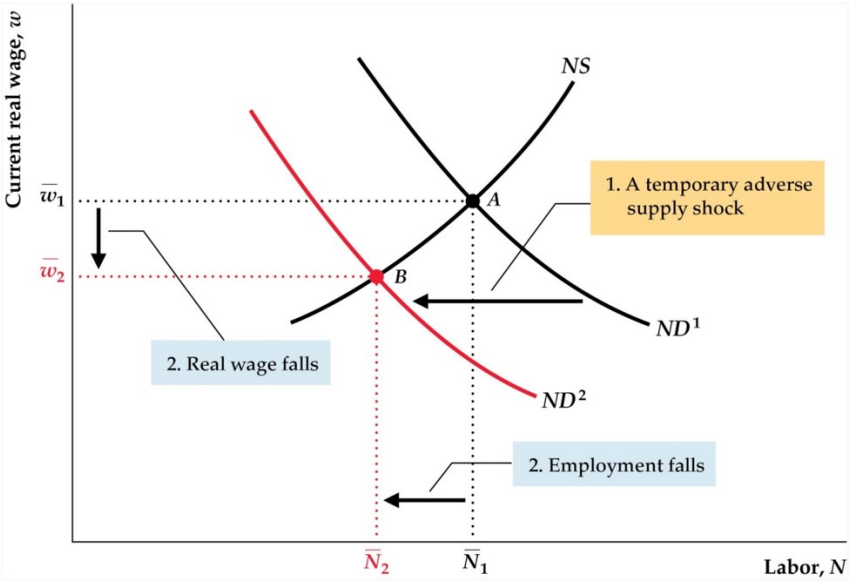


(Classical) Labor Market Equilibrium

Why "classical"?

- Quantity of labor and real wage fully determined by a labor demand curve and a labor supply curve
- Assumes that wages and quantities respond quickly to shocks
- Workers are always responding optimally to the prevailing real wage $\Rightarrow$ there is no notion of unemployment
  - Everyone is working the exact number of hours they want to work at the equilibrium real wage
- In practice, wages may take time to adjust in response to shocks, and some people who would like to work at the current wage may not find a job

Labor Demand Shock



Full Employment

- The classical model is better suited to study **full employment**
- This is the equilibrium level of employment $\bar{N}$ that arises in the classical model, where wages adjust instantly
- The level of output that is consistent with full employment is called **full employment output** or **potential output**

$$\bar{Y} = AF(K, \bar{N})$$

- It is the level of output that would be produced under full employment, i.e. if everyone who wanted to work could work the number of hours they wanted to at the equilibrium real wage $\bar{w}$

Full Employment

- Full employment $\bar{N}$ and potential output $\bar{Y}$ fluctuate with factors that shift labor demand and labor supply

1. Shocks to productivity A
2. Shocks to capital K
3. Shifts in labor supply

5. Unemployment

Unemployment

- Classical model: everyone who wants to work at wage rate $\bar{w}$ can find a job
- This is not how economies work in practice: often, many people would like to work at prevailing wages and cannot find a job
- Thus the economy's factors of production (specifically, labor) may be underutilized at times, and output may be below its potential level

$$N < \bar{N} \Rightarrow Y < \bar{Y}$$

- Recall the definition of an unemployed person:
  - Anyone who does not have a job but is looking for one

Measuring Unemployment

- Every month, the Bureau of Labor Statistics (BLS) surveys 60,000 households via the **Current Population Survey** (CPS)
- The BLS asks many questions about demographic characteristics and labor force status
- Every person who is 16 years of age or older is assigned to one of three categories:

1. Employed, if that person worked full- or part-time during the past week
2. Unemployed, if that person didn't work during the past week but looked for work in the past four weeks
3. Out of the labor force, if that person did not work during the past week and did not look for work during the past four weeks

Employment Status of US Adult Population in December 2024

Category	**Number (M)**	**% of labor force**	**% of adult pop.**
Employed workers, (a)	161.7	96%	60% (employment rate)
Unemployed workers, (b)	6.9	4% (unemployment rate)	3%
Labor force, $(c) = (a) + (b)$	168.6	100%	63% (participation rate)
Not in labor force, (d)	101.1		37%
Adult population, $(e) = (c) + (d)$	269.7		100%

Source: Bureau of Labor Statistics, Table A-1 of the Employment Situation release

Measuring Unemployment

Let

- E denote the number of employed people
- U denote the number of unemployed people
- NLF denote the number of people not in the labor force

$$\text{Adult Population} = E + U + NLF$$

- The **labor force** is the sum of employed and unemployed people.

$$LF = E + U$$

- The **participation rate** is the fraction of the adult population in the labor force

$$PR = \frac{LF}{\text{Adult Population}} = \frac{E + U}{E + U + NLF}$$

Measuring Unemployment

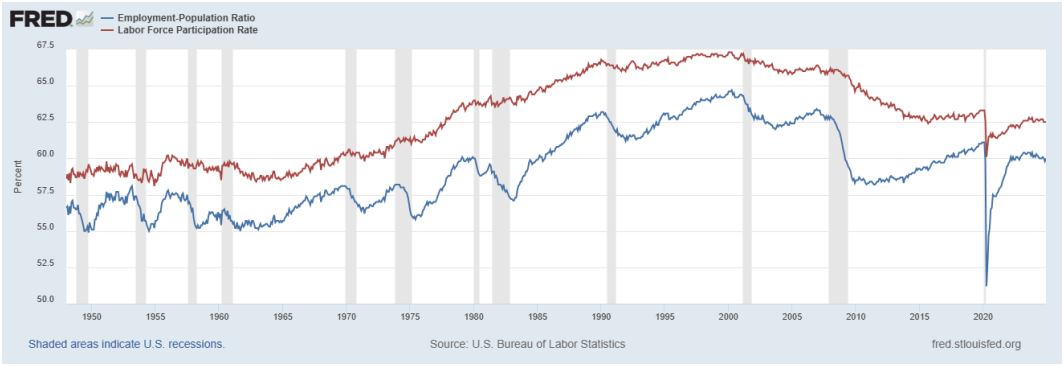
- The **unemployment rate** is the percentage of adults in the labor force that are unemployed

$$u = \frac{U}{LF}$$

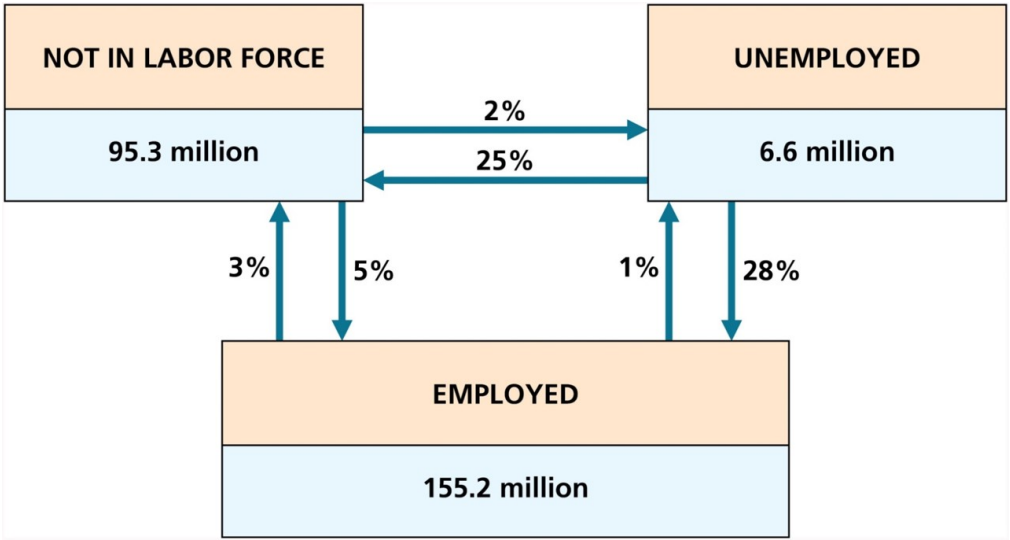
- The **employment rate** is the percentage of adults that are employed

$$EPOP = \frac{E}{\text{Adult Population}}$$

EPOP and LFP


FRED
Employment-Population Ratio
Labor Force Participation Rate
Percent
67.5
65.0
62.5
60.0
57.5
55.0
52.5
50.0
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
Shaded areas indicate U.S. recessions.
Source: U.S. Bureau of Labor Statistics
fred.stlouisfed.org

Labor Market Flows in May 2015



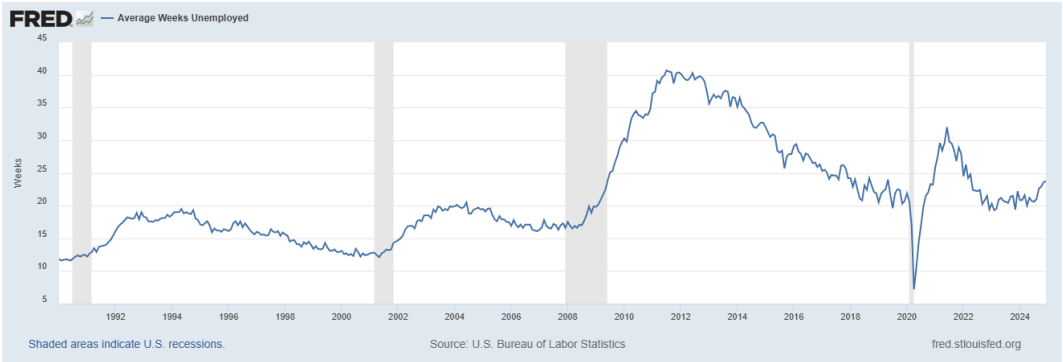
Unemployment Flows

- Some unemployed workers find jobs, others leave the labor force
- Some of the unemployed workers who leave the labor force are called **discouraged workers**
- They looked for jobs, but became discouraged and stopped searching
- Not all unemployed workers who leave the LF are discouraged: others simply retire or go to school

Unemployment Duration

- The period of time during which a person is continuously unemployed is called an **unemployment spell**
- The duration of unemployment is the length of unemployment spells and is of particular interest to economists
  - When unemployment duration is low, this means that unemployed workers are able to find new jobs relatively quickly
  - When unemployment duration is high, this means that it takes a long time for unemployed workers to find new jobs.
- Long unemployment spells have many social costs: they force people to dissave, may lead to the erosion of skills/human capital, and may be costly for taxpayers
- Reducing unemployment duration is typically an important policy goal

Unemployment Duration



Why is unemployment never equal to zero?

- Unemployment typically rises during recessions
- But even during expansions the unemployment rate is positive
- There are two types of unemployment:
  1. **Frictional Unemployment** arises from the normal functioning of the economy. People quit jobs all the time to look for better jobs. The process of finding a new job ("matching") takes time for many reasons.
  2. **Structural Unemployment** refers to longer-term unemployment that does not necessarily arise due to the dynamism of the economy but rather due to long-term skill mismatches and others structural issues
- Frictional unemployment can be a sign of a healthy and dynamic economy, while structural unemployment generally reflects problematic social and economic issues.
- Factors such as deindustrialization and offshoring tend to contribute to structural unemployment

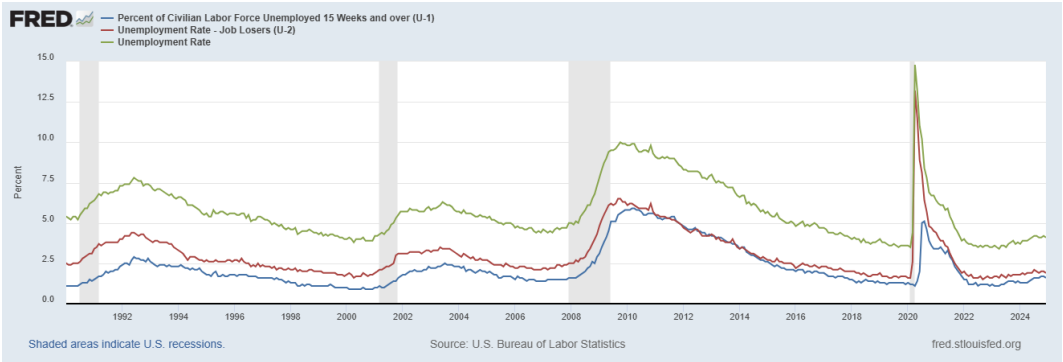
The Natural Rate of Unemployment

- The NRU, $\bar{u}$, is the unemployment rate that prevails when the economy is at full employment
- As output fluctuates around its potential level, so does the unemployment rate fluctuate around the NRU
- The difference between the unemployment rate and the NRU is called **cyclical unemployment**

$$u_t - \bar{u}_t$$

- Cyclical unemployment is positive during recessions and negative during expansions

Measures of Unemployment



6. Okun's Law

Output and Unemployment

- Most short-run fluctuations in output are associated with fluctuations in employment
- Productivity and capital are typically fixed in the short-run, meaning that deviations in output from its potential level are associated with deviations of unemployment from its natural level
- Okun's "Law" is an empirical relationship between the output gap and cyclical unemployment

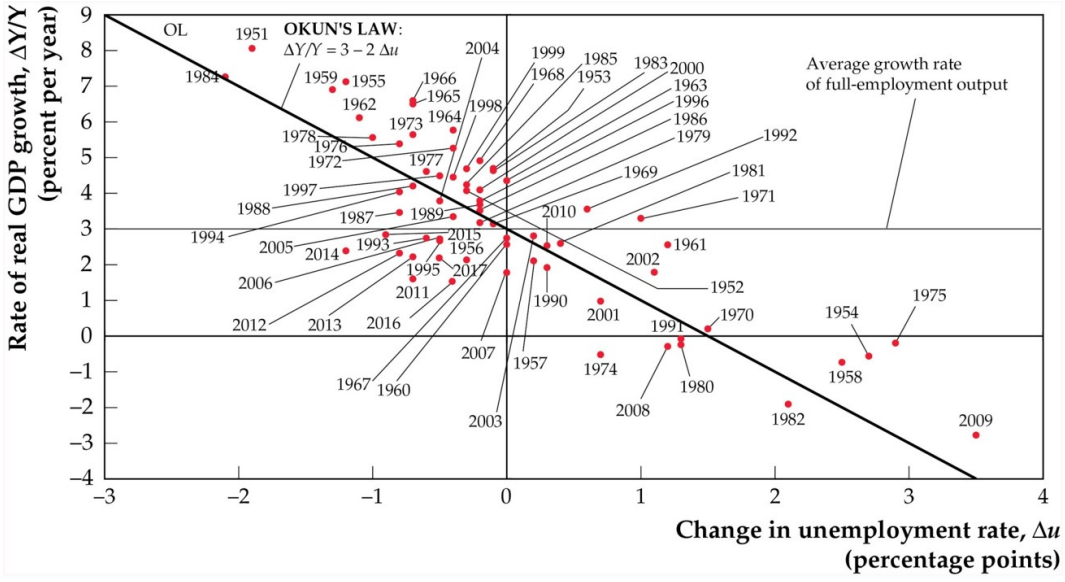
$$\frac{\bar{Y}_t - Y_t}{\bar{Y}_t} = 2 \times (u_t - \bar{u}_t)$$

- Named after Arthur Okun, chairman of the CEA in the 1960s, during LBJ's presidency
- The output gap increases by 2 percentage points for each percentage point of cyclical unemployment

Okun's Law

- Why is the coefficient equal to 2 and not 1?
- A rise in unemployment tends to understate the extent to which labor input is underutilized
- Labor force participation is **procyclical**: it rises when output is above potential, and falls when output is below potential
- Since the LFPR falls during recessions, movements in the unemployment rate understate the true change in hours worked/persons employed
- Recall that the denominator of the unemployment rate is the LF, not the adult population

Okun's Law


OKUN'S LAW:
$\Delta Y/Y = 3 - 2\,\Delta u$
OL
Average growth rate of full-employment output
Rate of real GDP growth, ΔY/Y (percent per year)
Change in unemployment rate, Δu (percentage points)
1951
1984
1959
1955
1962
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1965
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