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4th Firm Heterogeneity and Macroeconomics Workshop, Bonn

December 12, 2025

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  - Lending arrangements (i.e., evergreening, restructuring)



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- What are the effects of soft credit in a GE model of fluctuations and growth?

## This Paper: Overview

- 1. Develop a GE model with key features:
  - Aggregate fluctuations with tractable firm heterogeneity
  - Defaultable firm debt + endogenous firm exit
  - Endogenous productivity growth with innovation externalities

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  - Defaultable firm debt + endogenous firm exit
  - Endogenous productivity growth with innovation externalities
- 2. Calibrate the model
  - Key parameters estimated using micro data
- 3. Analyze impact of soft credit interventions
  - Evergreening [today], restructuring and fiscal transfers [in the paper]
  - Impact on balanced growth path, response to shocks & welfare
  - Comparative statics w.r.t. key parameters

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General equilibrium effects: more firms survive, creating labor market congestion

Higher wages compress firm profits, reducing investment (in capital and R&D)

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## Effects of eliminating soft credit (evergreening) in the U.S.:

- GDP growth ↑ by 11.5% (2%→2.23%)
- Firm exit, credit spreads, GDP volatility ↑
- Welfare gains of  $\sim$ 5% strongly dependent on strength of innovation externalities  $_{3/26}$

# Roadmap

Model

Characterization of the model

Calibration and estimation

Results

# Roadmap

Model

Characterization of the mode

Calibration and estimation

Result

#### **Environment**

Time is discrete and infinite t = 0, 1, ...

#### Demographics:

- $\bullet \quad \text{Representative } \textbf{household} \Rightarrow \text{consumes, works, deposits with intermediaries} \bullet \quad \textbf{details}$
- Continuum of risk-neutral financial intermediaries ⇒ take deposits, lend to firms
- Firms borrow, invest in capital and R&D, hire labor, and produce

Firms may default & exit, replaced by entrants with no capital ⇒ constant mass

# Firms: Technology

Decreasing returns to scale production function

$$y = (z\varepsilon)^{1-\eta} (\zeta k)^{\alpha} n^{\eta}, \quad \alpha + \eta < 1$$

- z is fundamental productivity (same for all firms in eq.)
- $\varepsilon \sim F[0,\infty)$  is idiosyncratic productivity, i.i.d. across time and firms
- ζ are AR(1) capital quality shocks [Merton, 1973; Brunnermeier & Sannikov, 2014]

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Firms choose z' subject to convex cost

$$R\&D = \phi \left( \underbrace{\frac{z'}{z^{1-\rho} \underbrace{(Z\varepsilon^*)^{\rho}}_{own}}}_{exerage} \right)^{\kappa}$$

where  $\rho$  disciplines learning externalities [Lucas & Moll, 2014]  $\Rightarrow$  estimate on micro data

#### **Firms: Financial Frictions**

1. Interperiod debt b' pays interest rate  $q^{-1}$ , subject to collateral constraint

$$b' \leq \theta k'$$

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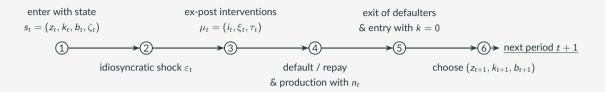
$$b' < \theta k'$$

2. Intraperiod debt  $\ell$  pays interest rate i, used to satisfy working capital constraint

$$\ell > wn$$

3. Limited liability: firms may default on intraperiod debt and exit

# Timeline within a period



#### Ex-post soft credit interventions $\mu$ :

- Evergreening: lenders may extend cheaper intraperiod credit i
- Restructuring: lenders may write off fraction  $\xi$  of existing debt b [not today]
- Fiscal support: govt may issue targeted transfer  $\tau$  [not today]

#### Firm Problem

1. Enter period with  $s \equiv (z, k, b, \zeta)$ , observe  $\varepsilon$  and i, choose to default or not:

$$V_0(s,\varepsilon) = \max\{V^p(s,\varepsilon),0\}$$

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2. If repay, pay fixed cost  $\nu$ , hire labor n, borrow  $\ell$ , produce:

$$V^{p}(s,\varepsilon) = \max_{n,\ell} (z\varepsilon)^{1-\eta} (\zeta k)^{\alpha} n^{\eta} - w \cdot n + \ell - (1+i)\ell - \nu - b + (1-\delta)\zeta k + V_{1}(s)$$
s.t.  $\ell > wn$ 

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s.t.  $\ell \geq wn$ 

3. Exiting firms replaced with entrants; all firms choose (z', k', b'):

$$V_1(s) = \max_{z',k',b'} -\phi \cdot \left[ \frac{z'}{z^{1-\rho} (Z(\varepsilon^*)^\rho)} \right]^{\kappa} - k' + q \cdot b' + \mathbb{E}[M' \cdot V_0(s',\varepsilon')]$$
  
s.t.  $b' \leq \theta k'$ 

#### Lenders: dynamic problem

- Each firm maintains a relationship with a risk-neutral lender
- Lender chooses deposits d' and price of debt q s.t. deposit funding constraint

$$W_1(s) = \max_{q,d'} \mathbb{E}\{M' \cdot [W_0(s', \varepsilon') - d']\}$$
  
s.t.  $qb' \leq Q^d \cdot d'$ 

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Free entry: zero expected profits

$$W_1(s) = 0 \Rightarrow q(s) = rac{\mathbb{E}[M' \cdot W_0(s', \varepsilon')]}{b'}$$

Ex-post profits/losses are rebated lump-sum to household

# Lenders: intraperiod problem

- Lenders raise resources at linear cost  $\omega$  to fund intraperiod lending
- Hard credit: lend at  $i = \omega$  to non-defaulting firms
- Evergreening: internalize effects of *i* on default decision:

$$\begin{split} W_0(s,\varepsilon) &= \max_{\substack{i \geq i^{\text{reg}} \\ i \geq i^{\text{reg}}}} \underbrace{\mathbf{1}[\varepsilon \geq \bar{\varepsilon}(s; i)]}_{\text{no default}} \{ \underbrace{b + (i - \omega) \cdot w \cdot n(s, \varepsilon; i)}_{\text{lend, continue}} \} \\ &+ \underbrace{\mathbf{1}[\varepsilon < \bar{\varepsilon}(s; i)]}_{\text{default}} \underbrace{(1 - \lambda)\zeta k}_{\text{recovery}} \end{split}$$

*i<sup>reg</sup>*: minimum rate set by regulation

# Roadmap

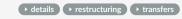
Mode

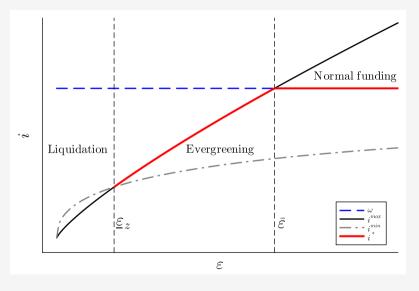
Characterization of the model

Calibration and estimation

Result

# Intraperiod equilibrium: evergreening





- Standard conditions for debt, capital
- Borrowing constraint always binding
- Innovation choice:

$$\frac{z'}{z} = \underbrace{\left(\varepsilon^*\right)^{\rho \frac{\kappa}{\kappa - \frac{1 - \eta}{1 - \alpha}}}}_{\text{externality}} \times \underbrace{\left\{\frac{\mathbb{E}\left[M \cdot \int_{\overline{\varepsilon'}}^{\infty} \frac{\partial V^{\rho}}{\partial z'}(s', \varepsilon') \mathrm{d}F(\varepsilon')\right]}{\kappa \phi}\right\}^{\frac{1}{\kappa - \frac{1 - \eta}{1 - \alpha}}}}_{\text{expected payoff relative to cost}}$$

where

$$\varepsilon^* \equiv \frac{\int_{\underline{\varepsilon}}^{\infty} \varepsilon dF(\varepsilon)}{1 - F(\underline{\varepsilon})}$$

is the average productivity of incumbent firms

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## **Calibration strategy**

- Calibration to annual U.S. data
  - Treat evergreening economy as the benchmark
  - Based on evidence in Faria-e-Castro, Paul & Sánchez (2024)

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- Calibration to annual U.S. data
  - Treat evergreening economy as the benchmark
  - Based on evidence in Faria-e-Castro, Paul & Sánchez (2024)
- Proceed in three steps:
  - 1. External calibration of standard parameters Odetails
  - 2. Direct estimation of parameters related to R&D cost and externality  $(\kappa, \rho)$
  - 3. Internal calibration of remaining parameters to match U.S. economy moments:

$$(\sigma_{\varepsilon}, \nu, i^{\mathsf{reg}}, \beta, \psi, \phi, \rho_{\zeta}, \sigma_{\zeta})$$
 details

#### Direct estimation of R&D parameters

• Firm i that wants to increase its productivity from  $z_{i,s,t}$  to  $z_{i,s,t+1}$  must spend

$$\mathrm{R\&D}_{i,s,t} = \phi_t \times \left(\frac{z_{i,s,t+1}}{z_{i,s,t}^{1-\rho}(x_{s,t}^*)^{\rho}}\right)^{\kappa},$$

where  $x_{s,t}^*$  is average productivity of industry s,  $\rho$  is learning weight,  $\kappa$  is cost curvature,  $\phi_t$  is common scale

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Rearranging terms & taking logs,

$$\log z_{i,s,t+1} - \log z_{i,s,t} = \frac{1}{\kappa} \cdot (\log R \& D_{i,s,t} + \log \phi_t) + \rho \cdot \log \frac{x_{s,t}^*}{z_{i,s,t}}.$$

# **Empirical specification & identification**

The empirical specification uses predetermined productivity terms and time fixed effects

$$\log z_{i,s,t+1} - \log z_{i,s,t-1} = \alpha_t + \beta_1 \log R \& D_{i,s,t} + \beta_2 \log \frac{x_{s,t-1}^*}{z_{i,s,t-1}} + u_{i,s,t}$$

• Endogeneity concern: R&D correlated with unobservables (e.g. expected growth)

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- **First stage**: higher credits  $\Rightarrow$  higher reported firm R&D; F-stat  $\approx$  15–20

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- Data: annual Compustat, sample: 1982–2023; obtain firm-level TFP estimates

**Results**: 
$$\hat{\beta}_1 = 0.33^{***}$$
,  $\hat{\beta}_2 = 0.30^{***} \Rightarrow \hat{\kappa} \approx 1/\hat{\beta}_1 \approx 3$ ,  $\hat{\rho} \approx 0.30$ 

• Robustness: (i) industry–time FE (absorbing  $w_{s,t}$ ), (ii) alternative TFP measure, (iii) economy-wide learning  $x_t^* \Rightarrow \beta_1 \in [0.33, 0.49], \beta_2 \in [0.30, 0.41]$ 

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ightharpoonup PD  $\epsilon$  Policy Functions ightharpoonup other soft credit

• Hard Credit:  $i^{reg} = \omega$ ; Evergreening:  $i^{reg} < \omega$ 

Moment	Hard Credit	Evergreening	
Subs. firm rate	0.00	5.95	
i, %	2.00	1.79	
Exit rate	9.77	4.99	
$arepsilon^*$	1.03	1.02	
GDP growth	2.23	2.00	
$\sigma(G_Y)$	3.12	2.44	
Spread, %	3.75	1.97	
K/Y ratio	2.04	2.36	
Wage	0.76	0.82	
CEV wrt HC	0.00	4.81	

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• Soft credit reduces firm exit and average prod. of incumbents  $\varepsilon^*$  ...

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... relative to hard credit economy, this reduces GDP growth & volatility

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Interperiod spreads decline which raises capital investment all else equal.

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• Evergreening distorts labor decision, raises wages, reduces incentives to invest

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▶ PD  $\epsilon$  ▶ Policy Functions

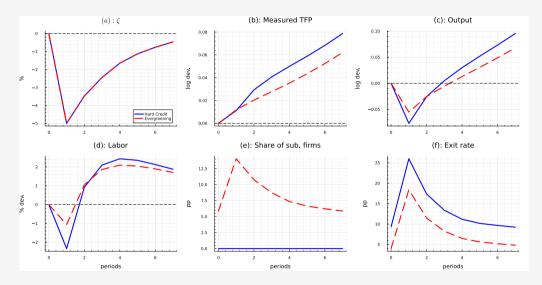
▶ other soft credit

• Growth effect dominates, soft credit reduces welfare

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### Soft credit moderates recessions [IRF to capital quality shock]

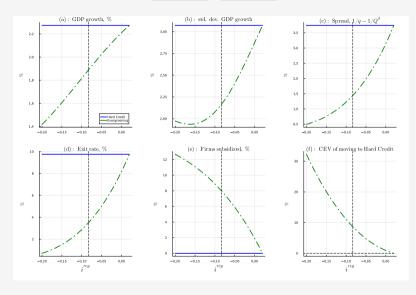




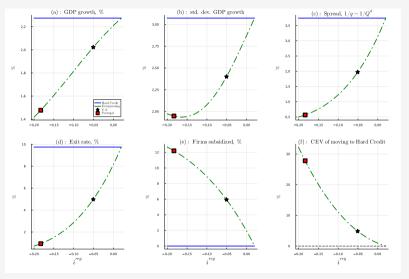
### Comparative statics w.r.t. *i*<sup>reg</sup>

▶ other policies

▶ misallocation



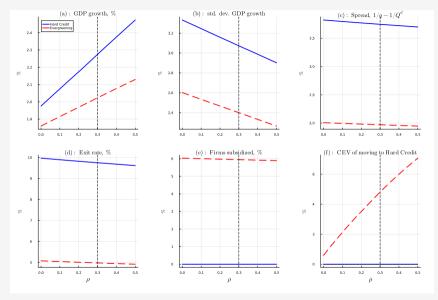
### Portugal vs. the US



Zombie share of 11.7% for Portugal estimated using the dataset of Albuquerque & Iyer (2024).

### Welfare effects of soft credit depend on $\rho$





• Research question: What are the effects of ex-post interventions ("soft credit") on business cycle fluctuations and growth?

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- Approach: Develop and calibrate tractable GE model with business cycles, heterogeneous firms, and endogenous growth.
- Key channel: soft credit allows low-productivity firms to survive longer, affecting other firms via innovation externalities and congestion in input markets
- This stabilizes the economy in the short run, but slows down long-run growth.
- Work in progress: (i) planner problem, (ii) monetary policy, (iii) recursive preferences, (iv) endogenous entry



- Banks may have incentives to keep distressed firms alive by rolling over credit
- Typically associated with aftermath of crises  $\rightarrow$  Japan in 1990s, Eurozone in 2010s
  - But: also prevalent among US banks recently [Faria-e-Castro, Paul, and Sanchez, 2024]
- Literature mostly empirical; welfare implications not clear
  - † prevents firm exit, stabilizes employment [Kashyap, Rajan & Stein, 2002]
    - does it make recessions less severe? ⇒ need a business cycle model
  - \$\rightarrow\$ linked to zombie lending [Caballero, Kashyap & Hoshi, 2008]
    - does it have long-run implications for productivity? ⇒ need a growth model
- Similarly, renegotiation (Chapter 11) allows for ex-post restructuring of debt

• Representative household maximizes PDV of period utility with discount factor  $\beta$ 

$$u(C_t, N_t) = \frac{C_t^{1-\sigma} \left[ 1 + (\sigma - 1) \chi \frac{N_t^{1+\varphi}}{1+\varphi} \right]}{1-\sigma}$$

- King, Plosser & Rebelo (1988) preferences required for balanced growth with constant hours
- Budget constraint:

$$C_t + Q_t^d D_{t+1} = w_t N_t + D_t + \Psi_t$$

- $\Psi_t$  are firm and intermediary profits
- Standard optimality conditions



- Binding working capital constraint  $\Rightarrow$  closed-form solutions for  $(n, \ell)$
- Operating profits linear in  $(z, \varepsilon)$ :

$$\max_{n,\ell \geq wn} (z\varepsilon)^{1-\eta} (\zeta k)^{\alpha} n^{\eta} - wn + \ell - (1+i)\ell \equiv \varepsilon \cdot \mathbf{z} \cdot \pi(\mathbf{k},\zeta;\mathbf{w},i)$$

Distress iff

$$V^p(s,\varepsilon) < 0 \Leftrightarrow \varepsilon < \bar{\varepsilon}$$

where

$$ar{arepsilon} = rac{b + 
u - (1 - \delta)\zeta k - V_1(s)}{z\pi(k,\zeta;w,i)} = ext{distress threshold}$$

Distress implies exit in the hard credit economy



### Define the following objects:

1.  $i^{\max}(s, \varepsilon)$ : max rate for which firm does not default

$$i^{\max}(s,\varepsilon) = \max\{i : \bar{\varepsilon}(s;i) = \varepsilon\}$$

2.  $i^{\min}(s,\varepsilon)$ : min rate for which lender is indifferent between lending or liquidating

$$i^{\min}(s,\varepsilon) = \min\{i : W_0(s,\varepsilon;i) = (1-\lambda)\zeta k\}$$

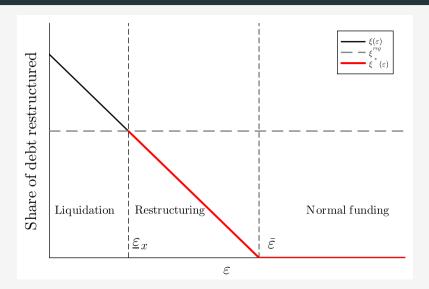
The intraperiod equilibrium can be characterized in terms of a <u>distress threshold</u>  $\bar{\varepsilon}(s)$  and a liquidation threshold  $\varepsilon_{z}(s)$  for idiosyncratic productivity:

1. If  $\varepsilon > \bar{\varepsilon}(s)$ , the firm is in the normal funding region and  $i = \omega$  (not distressed)

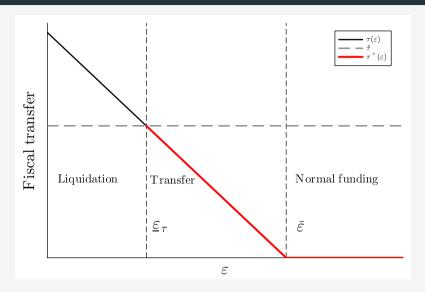
2. If  $\varepsilon \in [\underline{\varepsilon}_z(s), \bar{\varepsilon}(s)]$ , the firm is the evergreening region and  $i \in [i^{reg}, \omega)$  (distressed)

3. If  $\varepsilon < \underline{\varepsilon}_z(s)$ , the firm is liquidated.









The intraperiod equilibrium can be characterized in terms of a <u>distress threshold</u>  $\bar{\varepsilon}(s)$  and a liquidation threshold  $\underline{\varepsilon}_{x}(s)$  for idiosyncratic productivity:

1. If  $\varepsilon > \bar{\varepsilon}(s)$ , the firm is in the normal funding region (not distressed)

2. If  $\varepsilon \in [\underline{\varepsilon}_x(s), \overline{\varepsilon}(s)]$ , the firm is the restructuring region and  $x \in (0, \xi^{reg}]$  (distressed)

3. If  $\varepsilon < \underline{\varepsilon}_{x}(s)$ , the firm is liquidated.



A competitive equilibrium is a sequence of allocations

 $(C_t, N_t, D_{t+1}, z_{t+1}, k_{t+1}, b_{t+1}, n_t, \ell_t)$ , prices  $(w_t, q_t, i_t)$ , and thresholds for liquidation and distress  $(\underline{\varepsilon}_t, \overline{\varepsilon}_t)$ , such that for all t:

- 1. Households optimize over  $(C_t, N_t, D_{t+1})$ , taking prices and the distribution of firms as given
- 2. Firms optimize over  $(n_t, \ell_t, z_{t+1}, k_{t+1}, b_{t+1})$ , taking prices, lending policies, and thresholds as given
- 3. Lenders optimize over  $(q_t, i_t, \xi_t)$
- 4. Thresholds  $(\underline{\varepsilon}_t, \overline{\varepsilon}_t)$  satisfy the firm's indifference conditions implied by optimal lending contracts.
- 5. The labor, goods, deposit, intraperiod debt, and interperiod debt markets clear.

A BGP exists with detrending factor

$$z_t^{\frac{1-\eta}{1-\alpha}}$$

• We solve for a detrended version of the model, where all quantities are detrended:

$$x_t = \tilde{x}_t \cdot z_t^{\frac{1-\eta}{1-\alpha}}$$

• Letting  $G_z \equiv z_{t+1}/z_t$ , all real quantities in the economy grow at gross rate

$$G_z^{\frac{1-r_1}{1-c}}$$

# External calibration • Back



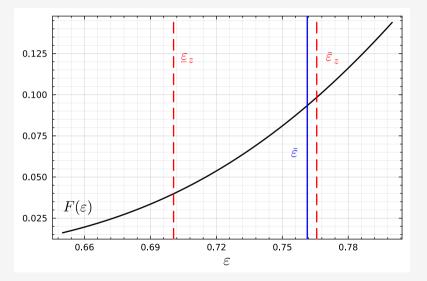
Parameter	Description	Value	Target/Reason
σ	Inverse EIS	1.5	-
$\varphi$	Inv. Frisch elasticity	1	Standard
$\alpha$	Capital share	$0.36  imes \psi$	Standard
$\eta$	Labor share	$0.64  imes \psi$	Standard
$\delta$	Depreciation rate	0.08	Standard
$\theta$	Collateral constraint	1	Debt to fixed-assets of 1 (FPS, 2024)
$\lambda$	Loss given default	0.35	Y-14 data
$\omega$	Lending cost	0.02	2% annual (Gilchrist & Zakrajsek, 2012)

### Internally calibrated parameters and shocks



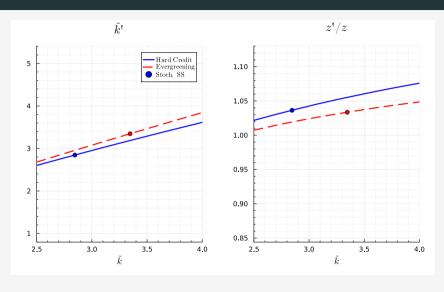
Parameter	Description	Value	Moment	Source	Data	Model
$\sigma_{arepsilon}$	Variance of prod.	0.192	TFP of exiting/continuing firms	Lee & Mukoyama (2015)	0.65	0.65
$\nu$	Fixed cost	0.012	Exit/default rate	Crane et al. (2022)	5.0%	5.0%
i <sup>reg</sup>	Lower bound int. rate	-0.051	Credit spread	Y-14 data	2.0%	2.0%
$\beta$	Discount factor	0.995	Real interest rate	U.S. data	3.0%	3.0%
$\psi$	Returns to scale	0.864	EBITDA/Value Added	Compustat	42.2%	41.6%
$ ilde{\phi}$	Level cost of R&D	0.171	GDP pc growth	U.S. data	2%	2%
$ ho_{\zeta}$	Capital quality persistence	0.685	GDP growth rate	U.S. data	0.044	0.041
$\sigma_{\zeta}$	Capital quality volatility	0.020	GDP growth rate	U.S. data	0.023	0.024





### Firm policy functions for physical capital and R&D



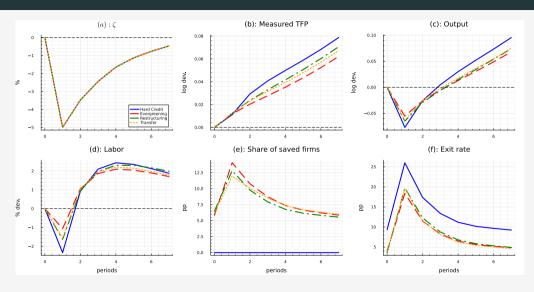




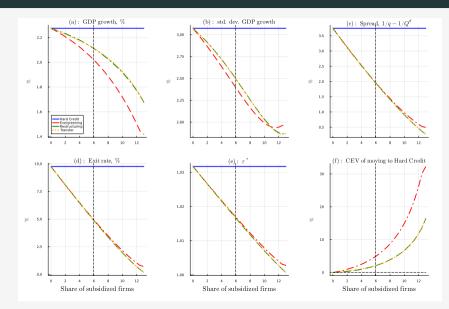
Moment	Hard Credit	Evergreening	Restructuring	Transfer
Subs. firm rate	0.00	5.95	5.96	5.96
Exit rate	9.77	4.99	4.92	4.94
$arepsilon^*$	1.03	1.02	1.02	1.02
GDP growth	2.23	2.00	2.08	2.09
$\sigma(G_Y)$	3.12	2.44	2.54	2.54
$\sigma(G_C)$	2.42	2.37	2.40	2.39
$\sigma(\log N)$	1.92	1.59	1.69	1.60
Spread, %	3.75	1.97	1.96	1.96
i, %	2.00	1.79	2.00	2.00
K/Y ratio	2.04	2.36	2.36	2.36
Wage	0.76	0.82	0.82	0.82
CEV wrt HC	0.00	4.81	2.02	2.05

### Capital quality shock: all economies









### **Evergreening vs. Restructuring/Transfers**



- For the same % of subsidized firms, evergreening provides more stabilization but reduces growth by more
  - Evergreened firms benefit from lower *i*
  - This raises their labor demand
- Low productivity firms demand relatively more labor
- Evergreening generates static misallocation
- Restructuring and transfers do not distort static labor choice

### Comparative statics w.r.t. $\rho$ , all policies



