

A Quantitative Theory of Relationship Lending

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What are the macro effects of relationship lending?

Fact: **lending relationships** are important in many loan markets

▶ figure

- price dispersion, sourcing persistence, “lifecycle” of spreads

Model: (i) multiple lenders + sourcing adjustment costs \implies relationships
(ii) financial frictions interact with implied imperfect competition

Estimate: directly estimate key demand parameters on Y-14Q micro-data

Validate: model matches lender switching, price evolution moments from data

Applications: equilibrium effects of changes in relationship strength / duration, impacts on aggregate dynamics

- Experiments (for now): financial crises (GFC 08); confidence crises (SVB 23); monetary policy transmission

Borrowers care which banks charge which prices \implies **2-tier demand**

1. **bank level:** bank j 's relationship intensity, spread over average IR
2. **aggregate:** joint distribution of relationships and spreads

Banks internalize relationship formation \implies **dynamic pricing**

- relationships induce heterogeneous actions *beyond financial conditions*

Key outcome: stark changes to net worth dynamics, plays out in several ways:

- *life cycle:* young / early relationship banks price below market
- *bank level:* expend relationship capital to smooth out shocks
- *aggregate:* slow moving net worth \rightarrow impact depends on shock

Stationary model overview

Time is discrete and infinite, $t = 0, 1, 2, \dots$

1. **rep. borrower:** demands loans *from all banks* to finance production
2. **heterogeneous banks:** finance loans subject to “standard” frictions
 - differentiation through relationship intensity \implies **market power**

Loan market: endogenous joint dist. of prices q and relationships s , $\mu(q, s)$

- *exogenous:* risk free rate $\bar{r} = \bar{q}^{-1} - 1$, wage \bar{w} , deposit price \bar{q}^d

Banks' problem

States: net worth n , customer capital s , return shock z

$$V(n, s, z; \mu) = \max_{q, e, n', \ell', d', s'} e + \bar{q}\pi \mathbb{E}_{z'} [V(n', s', z'; \mu)]$$

subject to:

[budget] $q\ell' + \psi(e) \leq n + z + \bar{q}^d d'$

[net worth dynamics] $n' = \ell' - d'$

[capital requirement] $\chi q\ell' \leq q\ell' - \bar{q}^d d'$

[loan demand] $\ell' = \ell'(q, s)$

[relationship formation] $s' = \rho_q \frac{q\ell'}{L'(\mu)} + \rho_s s$

- Relationship intensity as dynamic endogenous state (Gourio and Rudanko, 14)
- Affects loan pricing [▶ details on loan pricing](#)

Representative firm / borrower and loan demand

DRS production + working capital constraint on wages \implies loan demand

Borrow (in principle) from **all banks** $j \in [0, 1]$, choose sourcing given:

- q_j : loan price offered by j (implies interest rate $r(q_j)$)
- s_j : (relative) relationship with $j \rightarrow$ weighted average of past loan shares
- $\mu(q, s)$: joint distribution of prices and habits
 - borrower does not internalize current loan choices on $\{s'\}, \mu'$
 \implies “external” in the spirit of Ravn, Uribe, and Schmitt-Grohe (06)

Loan share adjustment subject to quadratic costs with level ϕ

- aggregate contraction not “costly by construction”

Borrower problem with relationships and adjustments

$$W(\mathcal{L}; \mu) = \max_{n, L', \mathcal{L}' = \{\ell'(q, s)\}} \underbrace{zn^\alpha - \bar{w}n}_{\text{op. profits}} + \underbrace{L' - \int \ell(q, s) d\mu(q, s)}_{\text{borrowing, net repayments}} - \underbrace{\frac{\phi}{2} L' \int \left(\frac{q\ell'(q, s)}{L'} - 1 - (s - S) \right)^2 d\mu(q, s)}_{\text{adjustment costs}} + \bar{q} \mathbb{E} [W(\mathcal{L}'; \mu')]$$

subject to:

[working cap.]

$$L' \geq \kappa \bar{w} n$$

[sourcing]

$$\int q\ell'(q, s) d\mu(q, s) \geq L'$$

2-part equilibrium loan demand system

1. Bank-specific loan demand

$$\underbrace{\frac{q\ell'(q, s; \mu)}{L'(\mu)}}_{\text{relative loan demand}} = 1 + \underbrace{(s - S)}_{\text{relationship shifter}} - \underbrace{\frac{\bar{q}}{\phi}(r(q) - R(\mu))}_{\text{elasticity} \times \text{IR spread}}$$

2. Aggregate loan demand

$$L'(\mu) = \kappa \bar{w} \left[\frac{\alpha z / \bar{w}}{1 + \kappa (\bar{q} \tilde{R}(\mu) - 1)} \right]^{\frac{1}{1-\alpha}}$$
$$\underbrace{\tilde{R}(\mu)}_{\text{"effective" IR}} = \underbrace{R(\mu)}_{\text{avg. IR}} + \underbrace{\mathbb{C}_{\mu}(r, s)}_{\text{cov. term}} - \underbrace{\frac{1}{2} \frac{\bar{q}}{\phi} \mathbb{V}_{\mu}(r)}_{\text{var. term}}$$

Equilibrium

A **stationary recursive competitive equilibrium** in this model consists of:

- loan demand functions $\ell'(q, s; \mu)$ and $L'(\mu)$;
- bank policies $g_q(n, s, z; \mu)$ and $g_d(n, s, z; \mu)$;
- distribution of prices and relationships $\mu(q, s)$; and
- distribution of bank states $m(n, s, z; \mu)$

which satisfy (i) borrower optimality; (ii) bank optimality; (iii) stationarity of bank state distribution m given policies g ; and (iv) **consistency of distributions m and μ given policies g** :

$$\mu(q, s) = \int \mathbf{1}[q = g_q(n, s, z; \mu)] m(\mathrm{d}n, s, \mathrm{d}z) \text{ for all } q, s$$

Strategy for quantifying the model

1. **externally assign** subset of parameters

- risk-free rate $\bar{r} = 2\%$, CR $\chi = 8\%$, bank failure rate $1 - \pi = 0.72\%$

2. **directly estimate** key relationship parameters ϕ , ρ_s , and ρ_q

- ϕ : Y-14Q measurements + model-implied demand curve + IV for supply
- ρ_s, ρ_q : use results above to measure s , estimate accumulation process

3. **internally calibrate** 4 remaining parameters to bank financing / pricing and aggregate debt moments ▶ internal calibration

Goal: tie our hands on ϕ , ρ_s , and ρ_q using semi-structural approach on micro data (2), then match other key features of banking industry (3).

Estimating ϕ : bank-specific demand curves

[details](#)

Idea: estimate model-implied demand on Y-14Q data

$$\frac{\ell_{fbt}}{L_{ft}} = \underbrace{\alpha_{fb} + \alpha_t + \Gamma X_{bt}}_{\text{FEs and controls}} + \underbrace{\beta(r_{fbt} - r_{ft})}_{\text{spread term}} + \underbrace{u_{fbt}}_{\text{s term}}$$

- ℓ_{fbt} : firm f loans from bank b at time t
- r_{fbt} : interest rate

Issue: instrument for spread term (simultaneity).

Solution: follow Khwaja and Mian (08), estimate

$$r_{fbt} - r_{ft} = \gamma_{ft} + \gamma_{bt} + v_{fbt}$$

- use $\hat{\gamma}_{bt}$ to instrument spread term
- measures pure credit supply shock

firm unit	TIN [1]	ISL [2]
$\hat{\beta}$, spread	-12.22*** (3.91)	-11.39* (5.90)
N obs.	119,394	406,054
implied $\hat{\phi}$	0.081	0.087

- TIN: individual firm
- ISL: Ind.-Size-Location
- $\hat{\phi} = -\bar{q}/\hat{\beta}$, \bar{q} consistent with

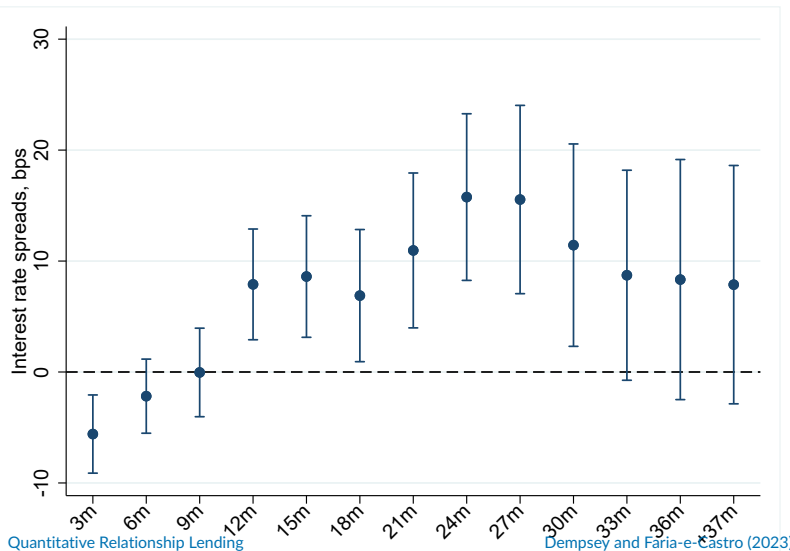
Estimating ρ_s and ρ_q : bank-level dynamics

Idea: use residuals \hat{u}_{fbt} to proxy s_{fbt} , then estimate law of motion via OLS:

$$\hat{u}_{fbt} = \alpha_{fb} + \alpha_t + \underbrace{\rho_q \frac{\ell_{fbt}}{L_{ft}}}_{\text{loan term}} + \underbrace{\rho_s \hat{u}_{fbt-1}}_{\text{lag term}} + \nu_{fbt}$$

firm unit	TIN [1]	ISL [2]
$\hat{\rho}_q$, lending	0.72*** (0.00)	0.75*** (0.00)
$\hat{\rho}_s$, persistence	0.24*** (0.00)	0.19*** (0.00)
<i>N</i> obs.	90,547	286,361

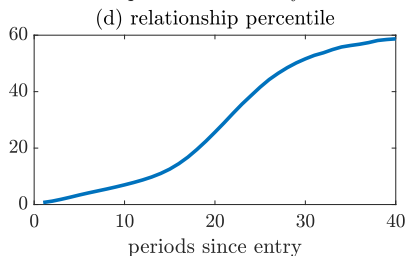
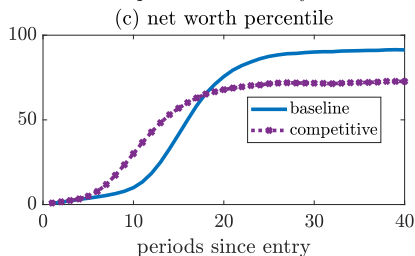
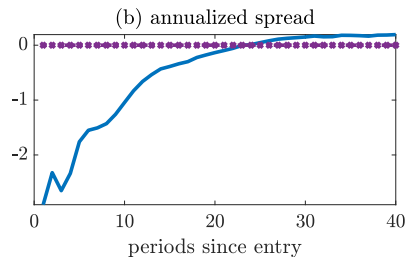
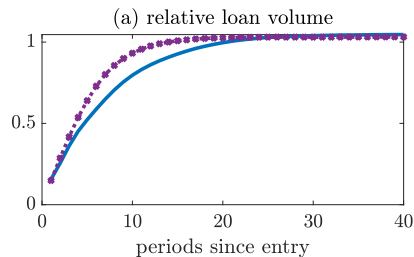
► magnitudes



Exercise: match similar loans in Y-14Q, compare terms for switching and non-switching

1. “honeymoon:” upon switching banks, firms pay lower interest rates
2. “holdup:” over time with bank, firms end up paying higher rates

Model validation: relationship lifecycle in the model



Additional facts on relationships and banking

Fact 1: Loan spreads widen over relationship (above)

- pronounced life cycle, ≈ 3 -5 years to reach avg. n , s , loan volume

Fact 2: Switching lenders is relatively infrequent

[▶ details](#)

- empirically, switches account for $\approx 3\%$ (40%) of total (new) loans
- model: measure switch as $\max \left\{ \frac{q\ell'}{L'} - s, 0 \right\} > 0$, share = 7.3%

Fact 3: Loan markets are highly concentrated

[▶ details](#)

- median loan market Herfindahl index $\approx 30\%$; model much lower
- *issue*: distribution of banks fairly compressed

[▶ details](#)[▶ distributions](#)

Pricing outcomes across model variants

		baseline	competitive	% diff rel to base
effective IR (pp, ann.)	\tilde{R}	4.69	3.24	-31.0
= average rate	R	4.53	3.24	-28.5
+ covariance term	$\mathbb{C}_{\mu}(r, s)$	0.08	-	-
+ variance term	$\mathbb{V}_{\mu}(r)$	-0.01	-	-
loan-weighted avg. IR	\bar{R}_L	4.59	3.24	-29.6
loan volume	L'	0.296	0.300	1.4

Higher effective IR, mostly driven by average rate.

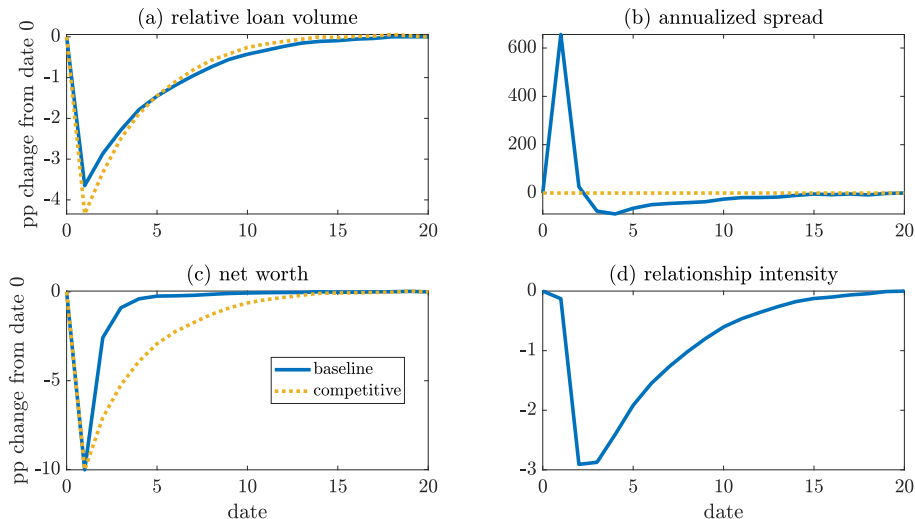
Banking industry moments across model variants

	baseline	competitive	% diff rel. to base
average net worth	0.029	0.027	-3.7
standard deviation, net worth	0.005	0.005	1.3
standard deviation, relationships	0.176	-	-
correlation, net worth and spread	0.10	0.00	-100.0
correlation, relationships and spread	0.12	-	-
correlation, net worth and relationships	0.97	-	-

- high corr. between n and s transmits to spreads, tempered by persistent z shocks
 - ρ_q critical for this effect
- more competitive model \implies less net worth on average (“franchise value effect”)

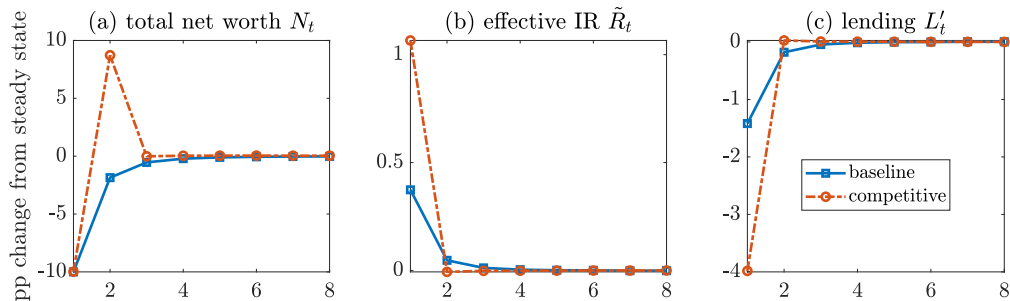
Idiosyncratic shock to net worth of one bank

► s shock



Experiment: wipe out 10% of n at date 1 at bank with $(n, s) = (\bar{n}, \bar{s})$ from indicated economy

Dynamic experiment 1: aggregate bank net worth shock



Shock: wipe out 10% of net worth at each bank

- more persistent drag on net worth, interest rates, lending and dividends
- why? mute price response on impact to preserve CC for future

Idiosyncratic vs. aggregate shocks to net worth

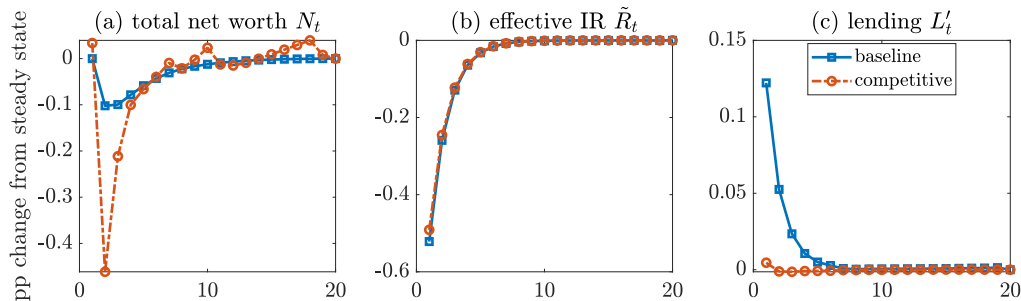
Idiosyncratic shock: *individual banks recapitalize faster* in our baseline model than the competitive model

Aggregate shock: *banking sector recapitalizes more slowly*

Why do net worth responses across models “switch” when moving from idiosyncratic to aggregate shocks?

- less n , more elastic lending in competitive case (no individual IR margin)
- widespread spread hikes push up R , limit “price punishment” in baseline
- drop in L' makes it less costly for banks to maintain loan share
- combined: the dynamic cost of hurting relationships looms relatively large

Dynamic experiment 2: real interest rate shock



Shock: drop \bar{r} from 2% to 0%, persistence of $\rho_{\bar{q}} = 0.5$

- allows substitution into external financing, \downarrow need for internal funds $\Rightarrow n \downarrow$
- strongest response in baseline model \rightarrow implications for MP transmission?

Conclusion and future directions

Model: imperfect competition via relationships + financial frictions

- **CC** \implies today's pricing decisions affect tomorrow's loan demand
- **frictions** \implies banks can expend CC to smooth shocks
- aggregate demand depends on joint distribution of prices and relationships

Quantitative analysis: estimate demand parameters using micro-data

- **cross-section:** endogenous life cycle, corr. b/w net worth, markups, CC
- **dynamics:** sluggish recovery, muted impact, greater persistence

On deck: hone in on validation, then study dynamics and implications for financial stability

Thank you!

We combine insights from 2 main literatures:

1. **financial accelerator / banking frictions:** e.g. BGG (99), Kiyotaki and Moore (97), Corbae and D'Erasmus (21)
 - novel competition structure with long-horizon pricing
 - heterogeneous bank “block” integrates with economy-wide loan market
2. **customer capital / habits:** e.g. Gourio and Rudanko (14), Ravn et al (06)
 - banks internalize habit formation, relationships pin down demand elasticity

towards a quantitative framework with credit market relationships.

- **empirics:** e.g. Rajan and Petersen (94), Atkeson et al. (19), Drechsler et al. (17)
 - **examples:** asymmetric information (Berger & Udell (1995), Berlin & Mester (1999)), hold-up costs (Ioannidou & Ongena, (2010)), imperfect competition: (Petersen & Rajan (1995))
- **equilibrium models:** e.g. Boualam (18), ...

Banks' loan pricing Euler equation slide:bank-problem-mainback

$$\underbrace{\Pi_t + \overbrace{\bar{q}\pi\rho_q\mathbb{E}_t \sum_{i=1}^{\infty} (\bar{q}\pi(\rho_q + \rho_s))^i \Pi_{t+i}}^{\text{"}\rho\text{ term"}}}_{\text{discounted lifetime net profits}} = \underbrace{\overbrace{\epsilon^{-1}(q\ell', q)}^{\text{"}\phi\text{ term"}}} \times \underbrace{\frac{\bar{q}}{q_t}\pi\mathbb{E}_t [(\psi^{-1})'(e_{t+1})]}_{\text{excess return (from today's market power)}}$$

$$\text{where } \Pi_t = \underbrace{\frac{\bar{q}}{q_t}\pi\mathbb{E}_t [(\psi^{-1})'(e_{t+1})]}_{\text{loan return}} - \underbrace{(\psi^{-1})'(e_t)}_{\text{funding cost}} + \underbrace{\lambda_t(1 - \chi)}_{\text{SV ease cap. req.}}$$

- Π_t : date t flow (economic) profits
- "fixed" CC ($\rho_q \rightarrow 0$) \implies static only $\implies \rho$ term $\rightarrow 0$
- no CC ($\phi \rightarrow 0$) \implies perfect competition $\implies \phi$ term and ρ term $\rightarrow 0$

► monopoly

► competitive

Evolution of bank distribution

Let the distribution of banks over states be denoted $m(x)$. This distribution evolves according to

$$T^* m(n', s') = \pi \int \mathbf{1} [n' = z' g_\ell(n, s) + g_a(n, s), s' = (1 - \rho)g_q(n, s)g_\ell(n, s) + \rho s] f(z') dm(n, s)$$

for continuing firms and

$$T^* m(x) = (1 - \pi)\bar{m}(x),$$

where $\bar{m}(x)$ is the distribution of entering banks (0 net worth, 0 customer capital)

[▶ Back](#)

- borrowers are indifferent about loan sourcing: care only about L'

$$L'(R) = \kappa w \left[\frac{\alpha/w}{1 + \kappa(\bar{q}R - 1)} \right]^{\frac{1}{1-\alpha}}$$

Note that this is the same as baseline with $R = \tilde{R}$

- banks choose ℓ' taking $q = 1/R$ as given:

$$\begin{aligned} V(n, z) &= \max_{e, \ell', d'} e + \bar{q}\pi \mathbb{E}[V(n', z')] \\ \text{subject to: [budget]} & \quad q\ell' + \psi(e) \leq n + z + \bar{q}^d d' \\ \text{[net worth dynamics]} & \quad n' = \ell' - d' \\ \text{[capital requirement]} & \quad \bar{q}^d d' \leq (1 - \chi)q\ell' \end{aligned}$$

Market power but no notion of customer capital

- aggregate demand same as competitive model (or baseline with $R = \tilde{R}$)

$$L'(R) = \kappa w \left[\frac{\alpha/w}{1 + \kappa(\bar{q}R - 1)} \right]^{\frac{1}{1-\alpha}}$$

- banks choose $q = 1/R$ taking $L'(q)$ as given:

$$V(n, z) = \max_{e, q, L', d', \delta'} e + \bar{q}\pi \mathbb{E}[V(n', z')]$$

subject to: [budget]

$$qL' + \psi(e) \leq n + z + \bar{q}^d d'$$

[net worth dynamics]

$$n' = L' + a$$

[capital requirement]

$$\bar{q}^d d' \leq (1 - \chi)qL'$$

[market power]

$$L' = L'(q)$$

	eqm policies	10% lending cut
adj. costs, median (n, s) bank (pp of lending)	0.004	0.370
adj. costs, aggregate (pp of total lending)	0.019	0.321

Adjustment costs on the order of

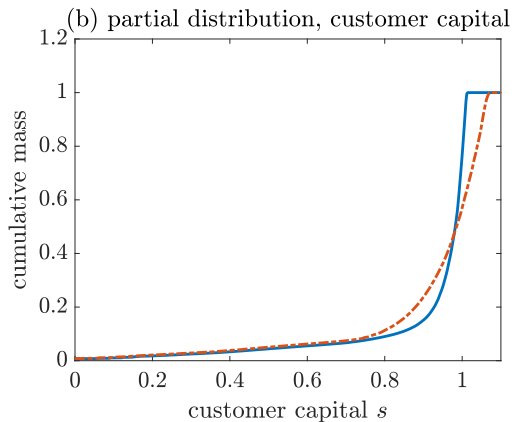
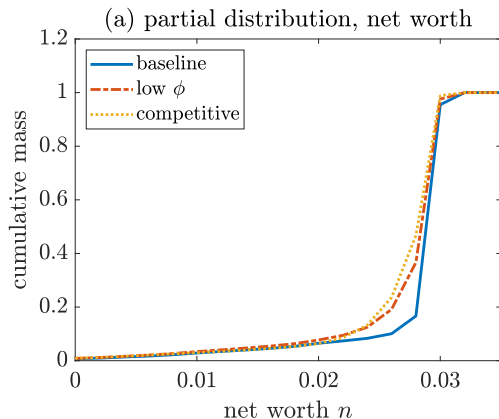
- 2 bps of total lending at equilibrium policies
- 32 bps of total lending with a 10% cut to these lending policies

Summary of calibration

[▶ back](#)

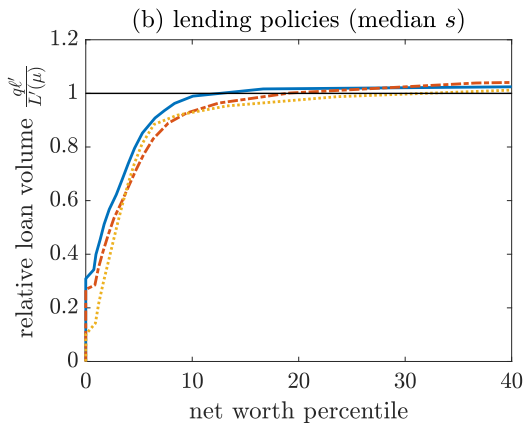
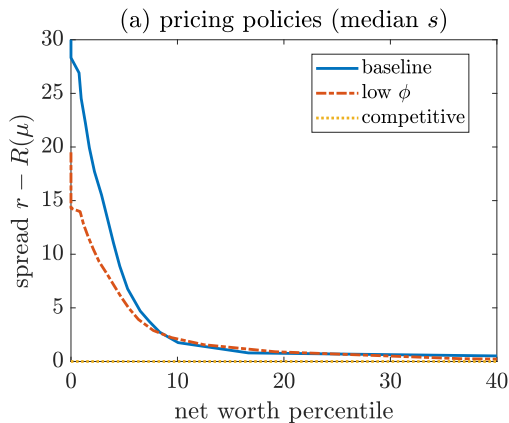
Description	Value	Target / Reason	Data	Model
Panel A: Externally Assigned Parameters				
\bar{q} discount factor	0.9951	annualized risk-free rate = 2%		
χ capital requirement	8%	Basel regulation		
π bank survival rate	0.9928	quarterly bank exit rate = 0.72%		
\bar{w} wage	1	normalization		
ν deposit liquidity premium	0.0004	annualized liq premium = 17 bps, $\bar{q}^d = \bar{q}(1 + \nu)$		
α returns to scale	0.75	profit share 20% - 30%		
Panel B: Directly Estimated Parameters				
ϕ lending share adj. costs	0.084			
ρ_s persistence of relationships	0.21	averages of estimates		
ρ_q lending effect on relationships	0.74			
Panel C: Internally Calibrated Parameters				
κ working capital constraint	0.958	business debt to GDP ratio	71.5%	71.4%
$\bar{\psi}$ equity issuance cost curvature	0.0094	gross equity issuance / NW	1.1%	1.1%
ρ_z persistence, net worth shocks	0.262	net dividend payouts / NW	5.8%	6.6%
σ_z std. dev., net worth shocks	0.0026	average net interest margin	3.3%	2.7%
		average bank leverage	91.5%	91.0%

Distributions across models



All models have lots of compression in both net worth and customer capital

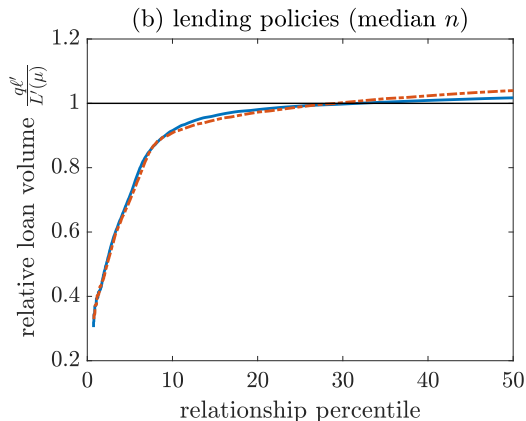
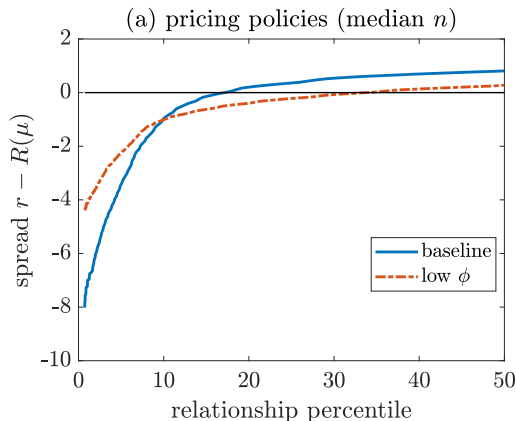
- low ϕ : more dispersion in both n (to left) and s distributions
- low ρ_q : harder to build up $s \implies$ more mass to left



Low $n \implies$ price “above market” to cut loan supply when net worth falls

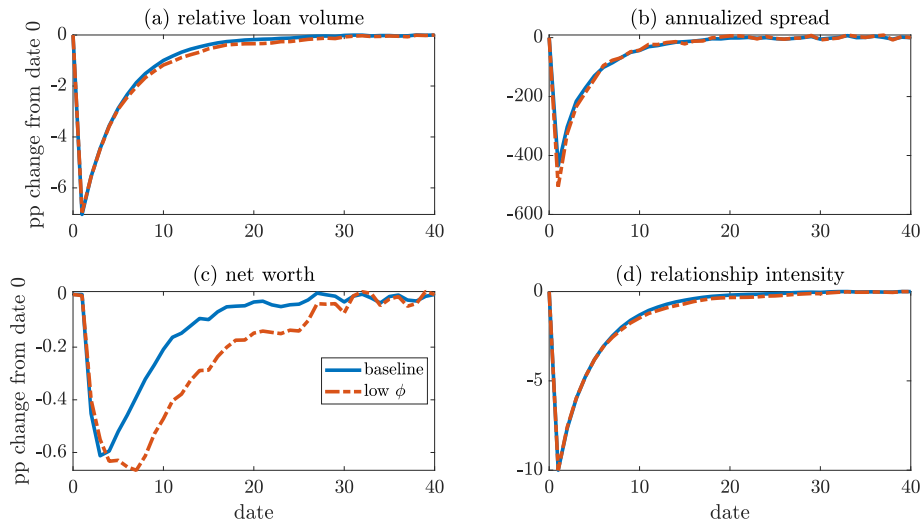
- effect strongest in least competitive models (high ϕ , low ρ_q)

Policies by relationship intensity



- banks with no existing relationships need to price to attract
- doesn't immediately translate into loan volume given demand system

An idiosyncratic shock to a bank's relationship

[▶ back to \$n\$ shock](#)

Experiment: wipe out 10% of s at date 1 at bank with $(n, s) = (\bar{n}, \bar{s})$ from indicated economy

Procedure: switching vs. non-switching loans

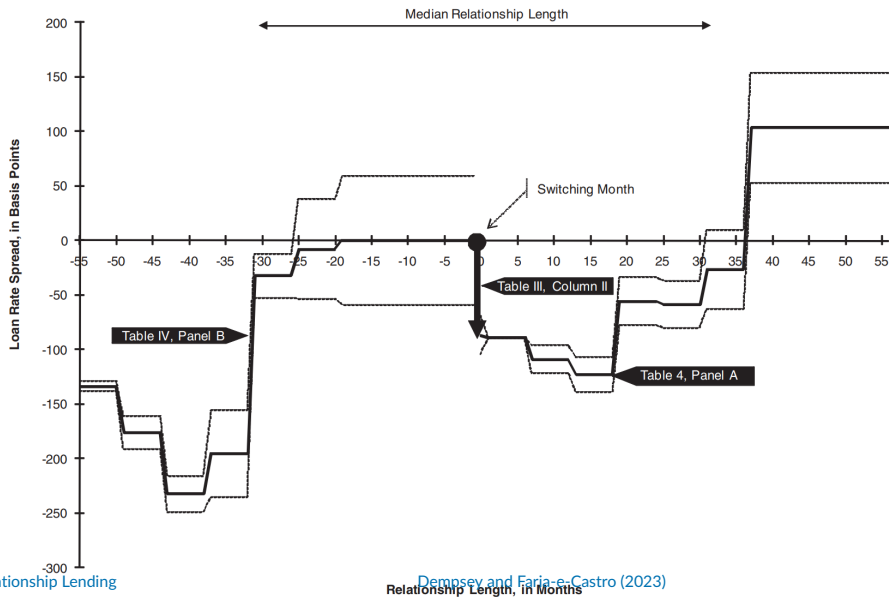
Goal: **match** switching vs. non-switching loans on a set of observables and compare spreads, following Ioannidou and Ongena (2010)

- 1. identify switches:** new loan from bank j from whom firm i has not borrowed in past $N = \{4, 8, 12\}$ quarters (may overstate: unbalanced panel, 1\$ M threshold, loan sales)
- 2. form matched pairs:** match switching and non-switching loans on: (i) quarter; (ii) bank; (iii) quarter of origination; (iv) loan maturity; (v) loan size (percentile); (vi) default probability (vigintile); (vii) loan type; (viii) variable v. fixed IR
 - robustness:* **tighter matching** on loan size, default probability, and add: (ix) NAICS 2; (x) firm size (percentile); and (xi) firm HQ CBSA
 - tradeoff:* tightness of matching vs sample size
 - more non-switches than switches \implies resample non-switches to pair each switch
- 3. compare spreads:** for each matched pair k , regress

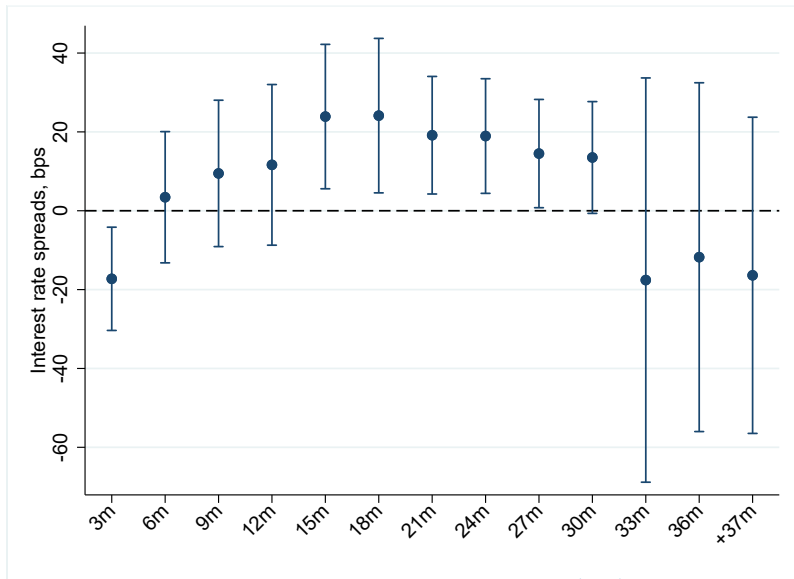
Quantitative Relationship Lending $\text{spread}_{kt} = \sum_{q=0}^Q \alpha_q \mathbf{1}[t = q] + \varepsilon_{kt}$ where q is time since switch

Ioannidou and Ongena (2010 JF) Figure 4

► back



Spreads after switching: tighter matching criterion

[▶ back](#)

Fact 1: switching is infrequent

[▶ back](#)

Source: Y-14Q. Switches defined in terms of number of loans.

Loan is a switch if it's (i) new and (ii) from a bank with which the firm has had no relationship in past 3 years

- definition follows Ioannidou & Ongena (2010)

Nature of the data \Rightarrow likely an upper bound:

- unbalanced panel: do not observe loans w/ balance < \$1M
- no small firms or small banks, where switching is less likely

Fact 2: loan markets are concentrated

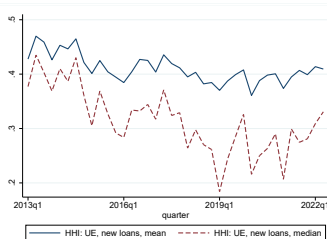
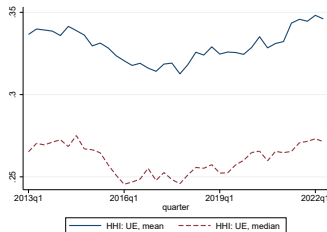
Compute Herfindahl-Hirschman Indices for local lending markets

- loan market defined as CBSA-quarter pair k
- The HHI is defined as

$$HHI_k = \sum_{i=1}^{N_k} \mu_{i,k}$$

where N_k is the number of banks present in market k and $\mu_{i,k}$ is the market share of bank i

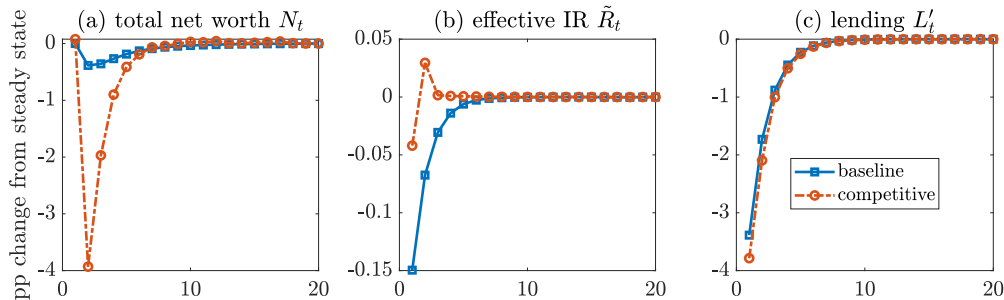
- The DOJ considers an industry with a HHI above 0.18 to denote a “highly concentrated industry”



Data: Y-14Q, schedule H.1

- Focus on new loans only (originated in the last 8 quarters)
- Criteria for inclusion:
 - Non-syndicated
 - Non-missing TIN with US address
 - Not in NAICS 52 (finance) or 92 (government)
 - Loan has non-negative interest rate and committed exposure
- Three definitions of a “firm”:
 1. Baseline: TIN
 2. Degryse et al. (2019): ISLT, quarter \times CBSA \times size decile \times 3-digit NAICS
- All standard errors are clustered at the BHC level
- X_{bt} are bank controls: size, leverage, loans/assets, deposits/liabs., liquidity

Dynamic experiment 2: aggregate TFP shock



Shock: 1% drop in TFP, persistence $\rho_z = 0.5$

- drop in demand for loans \implies net worth drops in both cases
- competitive: $n \downarrow$ causes constraints to bind and rates overshoot
- baseline: banks “fight” against drop by lowering rates further