The Cost of Capital and Misallocation in the United States

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The Cost of Capital and Misallocation in the United States

Goal: measure how dispersion in the cost of capital affects its allocation

Methodological contribution:

- Adapt a standard dynamic corporate finance model to enable measurement using micro data
- Derive a sufficient statistic for misallocation using credit registry data

Empirical Results (US):

- Low levels of misallocation in normal times ($\approx 0.5\%$ of GDP)
- Losses from misallocation increased to 1.1% of GDP in 2020-2021
- Possibly tied to mispricing of credit due to credit market interventions

Related literature

- **Measuring misallocation:**
  - Seminal work by Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
  - Contribution: use heterogeneity in funding costs to measure dispersion in MPK
- **Heterogeneity in the cost of capital:**
  - Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2021)
  - US: Gilchrist, Sim, and Zakrajsek (2013), David, Schmid, and Zeke (2022), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
  - Contribution:
    - Estimate firm cost of capital using credit registry data, correcting for maturity, default, etc.
    - Derive and estimate sufficient statistic for misallocation

Outline

1. Model
2. Welfare and misallocation
3. Measurement with credit registry data
4. Empirical results for the US

1. Model

Model

Borrowers 🏭

- Produce output $f(k, z)$
- Invest in capital k
- Long-term debt b
- Limited liability

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Lenders 💰

- Discount rate ρ
- Competitive pricing
- Recover ϕk in default

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Matching 🤝

- Borrower-lender match
- $\rho \sim$ match efficiency
- Heterogeneity in ρ

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Key question: how do heterogeneity in ρ and financial frictions distort the allocation of capital?

Firm problem

Value of repayment:

$$V(k,b,z) = \max_{k',b'} \pi(k,b,z,k',b') + \beta\mathbb{E}\overbrace{\left[\max\left\{V(k',b',z'),0\right\}\middle| z\right]}^{\text{Limited liability}}$$

Profits:

$$\pi(k,b,z,k',b') = f(k,z) + (1-\delta)k - k' - \theta b + Q(k',b',z)(b' - (1-\theta)b)$$

Price of debt:

$$Q(k',b',z) = \frac{\mathbb{E}\left[\mathcal{P}(k',b',z')(\theta + (1-\theta)Q(k'',b'',z')) + (1-\mathcal{P}(k',b',z'))\phi k'/b' \middle| k',b',z\right]}{\underbrace{1+\rho}_{\text{lender discount rate/match efficiency}}}$$

Firm cost of capital

Define the implicit interest rate paid by the firm as

$$1 + r_{i,t}^{firm} = \frac{\mathbb{E}_t \left[\mathcal{P}_{i,t+1} (\theta + (1-\theta) Q_{i,t+1}) \right]}{Q_{i,t}}$$

Lemma 1 (Firm cost of capital)

The firm cost of capital is:

$$1 + r_{i,t}^{firm} = \frac{1 + \rho_{i,t}}{1 + \Lambda_{i,t}} \qquad \Lambda_{i,t} \equiv \frac{\mathbb{E}_t \left[(1 - \mathcal{P}_{i,t+1}) \phi k_{i,t+1} / b_{i,t+1} \right]}{\mathbb{E}_t \left[\mathcal{P}_{i,t+1} \left(\theta + (1-\theta) Q_{i,t+1} \right) \right]}$$

▷ *Proof*

Λ: financial frictions wedge that arises due to limited liability and partial recovery ϕ

- $\phi = 0$: no recovery after default, then $r^{firm} = \rho$
- If $\phi > 0$, then $\Lambda > 0$ and $r^{firm} < \rho$: borrower only takes into account repayment states

Marginal revenue product of capital (MRPK)

$$\underbrace{(1 + r_{i,t}^{firm})\mathcal{M}_{i,t}}_{\text{cost of capital}} = \underbrace{\mathbb{E}_t[\mathcal{P}_{i,t+1}(f_k(k_{i,t+1}, z_{i,t+1}) + 1 - \delta)]}_{\text{expected marginal revenue product of capital}} \tag{1}$$

where $\mathcal{M}$ captures the price impact of the firm's actions

$$\mathcal{M} \equiv \frac{1 - \gamma \times \frac{b'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \frac{\partial \log Q}{\partial \log b'}} \qquad \gamma \equiv \frac{b' - (1-\theta)b}{b'}$$

- Heterogeneity in $r_{i,t}^{firm} \rightarrow$ heterogeneity in $MRPK_{i,t}$
- Approach: measure $r_{i,t}^{firm}$ by measuring $\rho_{i,t}$ and $\Lambda_{i,t}$

2. Welfare and misallocation

Aggregate economy and welfare

Decentralized Equilibrium:

$$Y^{DE} + (1-\delta)K^{DE} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}_{i,t+1}^{DE} \left(f(k_{i,t+1}^{DE}, z_{i,t+1}) + (1-\delta)k_{i,t+1}^{DE}\right) + (1-\mathcal{P}_{i,t+1}^{DE}) \cdot \phi k_{i,t+1}^{DE}\right] \mathrm{d}i$$

Aggregate economy and welfare

Decentralized Equilibrium:

$$Y^{DE} + (1-\delta)K^{DE} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^{DE}_{i,t+1}, z_{i,t+1}) + (1-\delta)k^{DE}_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1})\cdot \phi k^{DE}_{i,t+1}\right] \mathrm{d}i$$

Planner's problem:

- Inner problem: redistribute $\{k_{i,t+1}\}_i$ taking exit decisions and K^{DE} as given ▷ full planner problem
- Lower bound on full misallocation:

$$Y^* + (1-\delta)K^{DE} = \max_{\left\{k^*_{i,t+1}\right\}_i} \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^*_{i,t+1}, z_{i,t+1}) + (1-\delta)k^*_{i,t+1}\right)\right.$$
$$\left. + (1-\mathcal{P}^{DE}_{i,t+1})\cdot \phi k^*_{i,t+1}\right] \mathrm{d}i$$
$$\text{s.t.} \quad \int_0^1 k^*_{i,t+1}\mathrm{d}i = K^{DE}_{t+1} \equiv \int_0^1 k^{DE}_{i,t+1}\mathrm{d}i$$

Private vs. social optimality

Private optimality:

$$(1 + r_{i,t}^{firm})\mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$

Planner optimality:

- Define the social marginal product of capital at firm i, $r_{i,t}^{social}$

$$1 + r_{i,t}^{social} \equiv \mathbb{E}\left[\mathcal{P}_{i,t+1}^{DE}\left(f_k\left(k_{i,t+1}, z_{i,t+1}\right) + 1 - \delta\right) + \left(1 - \mathcal{P}_{i,t+1}^{DE}\right)\phi\right]$$

- Takes into account recovery in case of default

- Optimality: planner **equalizes** $r_{i,t}^{social}$ across firms at $\{k_{i,t+1}^{*}\}_i$

Misallocation

Proposition 1 (Misallocation)

Misallocation can be measured with $\mathbb{E}\left[r^{social}\right]$ *and* $Var\left(r^{social}\right)$ *as*

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{Var\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right] + \delta\right)^2}\right)$$

▷ *Proof*

- Extend Hughes and Majerovitz (2025) to a dynamic economy with default
- Set $\mathcal{E} = \frac{1}{2}$ and $\delta = 0.06$ ▷ calibration
- **Next:** we show how to measure $r_{i,t}^{social}$ using credit registry data

3. Measurement with credit registry data

Data: FR Y-14Q (Schedule H.1)

▷ summary stats. ▷ time series

- Quarterly loan-level panel on universe of loan facilities $> \$1M$
- Covers top 30/40 BHCs, 2014:Q4-2024Q4
- Detailed information on features of credit facilities
  - Origination date, size, maturity, interest rate/spread, probability of default, loss given default, fixed vs. floating, type of loan, etc.
- Focus on term loans issued to non-government, non-financial US companies

Pricing term loans

The break-even condition for a lender with discount rate ρ is

$$1 = \sum_{t=1}^{T} \left[\frac{P^t \mathbb{E}_0 \left[r_t \right] + P^{t-1} (1 - P) \left(1 - LGD \right)}{\left(1 + \rho \right)^t} \right] + \frac{P^T}{\left(1 + \rho \right)^T} \tag{2}$$

- T: maturity
- $\mathbb{E}_0[r_t]$: fixed interest rate or fixed spread over floating benchmark rate ▷ forward rates
- P: repayment probability (constant over time)
- LGD: loss given default (constant over time)
- $\Rightarrow$ Solve for lender's discount rate: ρ

Lender's discount rate

Fixed contractual rate:

Lemma 2 (Lender's discount rate)

For a fixed contractual rate loan:

$$1 + \rho = P(1 + r) + (1 - P)(1 - LGD)$$

▷ *Proof*

- ρ is independent of maturity T for fixed rate loans
- **Floating rate:** numerical solution of (2)

Firm cost of capital

Lemma 3 (Firm cost of capital)

We can solve for Λ *as*

$$\Lambda = \frac{(1-P)(1-LGD)}{1+\rho-(1-P)(1-LGD)}$$

and write the firm cost of capital as

$$1+r^{firm} = (1+\rho)-(1-P)(1-LGD)$$

▷ *Proof*

- $(1-P)(1-LGD) \simeq$ prob. of default event that does not result in a loss for the lender
- Measures pricing wedge between borrower and lender
- For fixed interest rate loans, use $(1+\rho)$ as in Lemma 2 to write $1+r^{firm} = (1+r)P$

Social cost of capital

Lemma 4 (Social cost of capital)

The social cost of capital can be written as:

$$
\begin{aligned}
1 + r^{social} &= (1 + r^{firm})\mathcal{M} + (1 - P)(1 - LGD)\,lev \\
&= \underbrace{(1+\rho)\,\mathcal{M}}_{\text{lender discount rate}} + \underbrace{(lev - \mathcal{M}) \cdot (1 - P) \cdot (1 - LGD)}_{\text{wedge due to financial frictions}}
\end{aligned}
$$

- social cost of capital $\simeq$ lender discount rate + wedge due to financial frictions

Sufficient statistic for misallocation

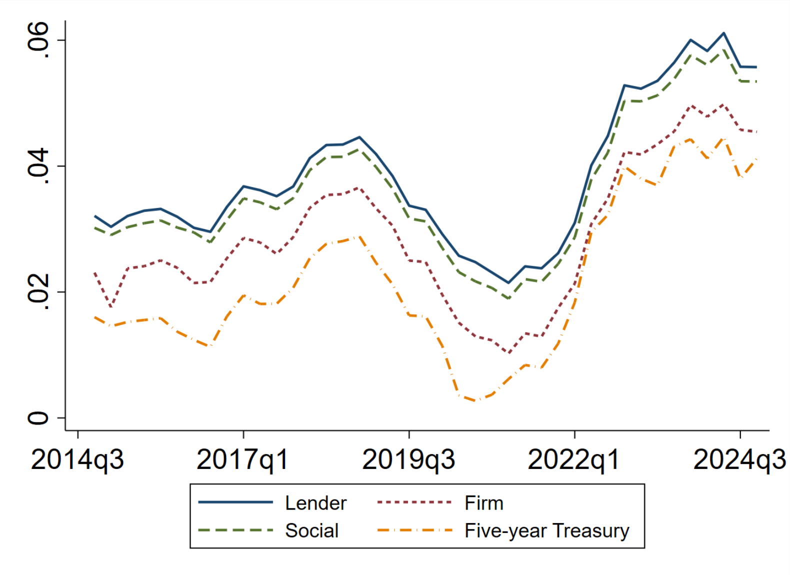
$$\log\left(Y^{*}/Y^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{\mathsf{Var}\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right]+\delta\right)^{2}}\right)$$

$$1+r_i^{social}=\left(1+\rho_i\right)\mathcal{M}_i+\left(lev_i-\mathcal{M}_i\right)\cdot\left(1-P_i\right)\cdot\left(1-LGD_i\right)$$

- Set $\mathcal{M}_i = 1$; reasonable approximation given our model ▷ Estimate $\mathcal{M}$
- Can measure misallocation directly with credit registry data!
- Dispersion in r^{social} comes from:
    1. Dispersion in lender's discount rate, ρ
    2. Dispersion in financial frictions wedge
    3. Covariance between ρ and financial frictions wedge

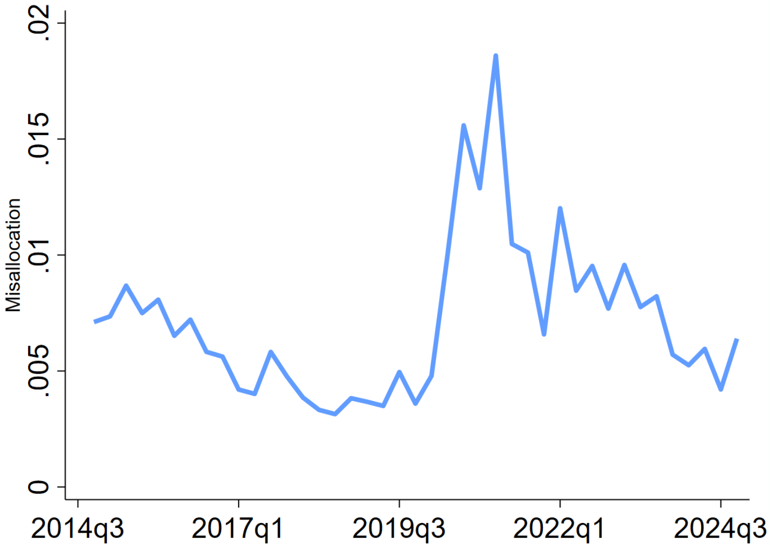
4. Empirical results

Average Discount Rate, Firm and Social Cost of Capital



- Rates follow 5y UST
- financial frictions
  $\Rightarrow r^{social} > r^{firm}$
- $\rho \approx r^{social} > r^{firm}$

Misallocation in the US, 2014-2024

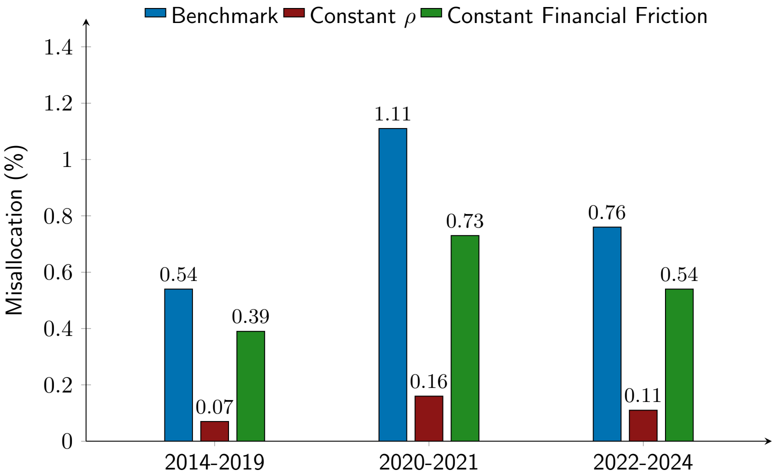


- About 0.5% before 2020
- ↑ to 1.1% in 2020-2021
- ↓ to 0.8% in 2022-2024

The 2020–2021 increase in misallocation

1. Predominantly explained by dispersion in ρ, rather than financial frictions wedge
2. Sharp rise in the coefficient of variation of ρ
3. Dispersion in ρ is traced to changes in the distribution of contractual rates (not P or LGD)
4. Driven by underpricing of very risky loans

1. The 2020-21 increase: sources of misallocation



- Mostly driven by heterogeneity in ρ
- Interaction between ρ and financial frictions $(0.54 > 0.07 + 0.39)$

$\Rightarrow$ 1. Predominantly explained by dispersion in ρ

▷ Decomposition

2. The 2020-21 increase: dispersion in ρ

- Heterogeneity in ρ is the most important driver of increase in misallocation during 2020-21

- As policy rates decreased in 2020-21, so did the mean ρ

- The standard deviation of ρ increased during this period

$\Rightarrow$ 2. Sharp rise in the coefficient of variation of ρ

3. The 2020-21 increase: role of contractual rates

- Approximate $\rho \approx r - (1 - P)LGD$
- The coefficient of variation depends on: (i) r, (ii) $(1 - P)LGD$ and (iii) their covariance

$$\frac{\mathbb{V}[\rho]^{0.5}}{\mathbb{E}[\rho]} \approx \frac{\left(\mathbb{V}[r] + \mathbb{V}[(1-P)LGD] - 2\,\mathbb{COV}[r,(1-P)LGD]\right)^{0.5}}{\mathbb{E}[r] - \mathbb{E}[(1-P)LGD]}$$

$\Rightarrow$ 3.Dispersion in ρ is traced to changes in the distribution of contractual rates (not P or LGD)

4. The 2020-21 increase: underpricing of risky loans

- Very risky loans—offered with unusually favorable contractual rates
- These loans have low implied ρ, increasing overall dispersion

Our hypothesis:

- Broad fiscal and monetary interventions (PPP, MSLP, PMCCF, SMCCF) supported distressed firms
- Lenders inferred explicit and implicit government guarantees for risky loans
- Moral hazard/zombie lending

Implication:

- Risk was mispriced, leading to credit misallocation
- Absent guarantees, risk would have been priced more accurately, improving allocative efficiency.

Cross-country comparison 🌐

▷ Assumptions

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
$\mu(r)$, %	78.7	14.1	83.0	16.8	3.9
$\sigma(r)$, %	38.1	2.9	93.3	5.2	1.5
$\mu(1-P)$, %	2.7	16.9	4.0	8.9	1.4
$\mu(1-LGD)$, % (World Bank)	42.8	42.8	18.2	63.9	81.0
Implied misallocation, %	4.9	2.2	21.5	1.7	0.6

- Developing countries: higher mean and standard deviation of contractual rates
- U.S.: lower mean and standard deviation of contractual rates, higher recovery
- Brazil: most extreme misallocation: 21.5%.
- Misallocation in the U.S. small but non-trivial: 0.6%.

Conclusions

- Develop a framework to measure misallocation using credit registry data
  1. Standard macrofinance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Inputs: standard credit registry variables (r, P, LGD, T, etc.)
- Application to U.S. credit registry data (FR Y-14Q)
  1. Estimate lender discount rates, firm-level cost of capital and social cost of capital
  2. Misallocation around 0.6% in normal times
  3. Sharp rise in 2020-21, possible tied to credit market interventions

Appendices

Proof: firm cost of capital

▷ back

$$\mathbb{E}_t\left[\frac{\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)}{Q_t}\right] = (1+\rho)\,\frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]+\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}$$

$$= (1+\rho)\left(1+\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}\right)^{-1}$$

$$= (1+\rho)\,(1+\Lambda)^{-1}$$

where

$$\Lambda \equiv \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}$$

Full planner problem

▷ back

$$U^* = \max_{\{\{k_{i,t}(S^{t-1}),\omega_{i,t}(S^t)\}_i\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(Y_t - I_t\right)$$

$$\text{s.t.} \quad \omega_{i,t}\left(S^t\right) \in \{0,1\}\, \forall i$$

$$\omega_{i,t+1}\left(S^{t+1}\right) \geq \omega_{i,t}\left(S^t\right)\, \forall S^t \subset S^{t+1}, \forall i$$

Can separate into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\left\{K_t,\{\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(\left(\max_{\left\{\{k_{i,t}(S^{t-1})\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} Y_t\right) - I_t\right)$$

Rewrite inner problem as:

$$Y_t^*\left(K_t, \{\omega_{it}\}_{i\in[0,1]}\right) = \max_{\{k_{i,t}\}_{i\in[0,1]}} \int_0^1 \mathbb{E}_{t-1}\left[\omega_{it} \cdot f\left(k_{it}; z_{it}\right) - \left(1-\omega_{it}\right) \cdot \left(\left(1-\delta\right) k_{it} - \phi\left(k_{it}\right)\right)\right] di$$

$$\text{s.t.} \quad K_t = \int_0^1 k_{it}\, di$$

Proof of Misallocation

▷ Back

- Formally, planner's problem is now the same as solving $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i)\mathrm{d}i$, where $f_i(k_i)$ is now expected output

- Apply Hughes and Majerovitz (2024), noting $\frac{dY}{dk} = r^{social} + \delta$

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right] + \delta\right)^2}\right)$$

- $\mathcal{E}$ is (negative) elasticity of output w.r.t. cost of capital $(r^{social} + \delta)$

Calibration of $\mathcal{E}$

▷ Back

- $\mathcal{E}_i$ is the elasticity of expected output with respect to the cost of capital
- Assume that $f(k, z) = z \cdot k^{\alpha}$ and there is no default, then

$$\mathcal{E} = \frac{\alpha}{1-\alpha}$$

- $\alpha = \frac{1}{3}$ implies $\mathcal{E} = \frac{1}{2}$

Summary Statistics

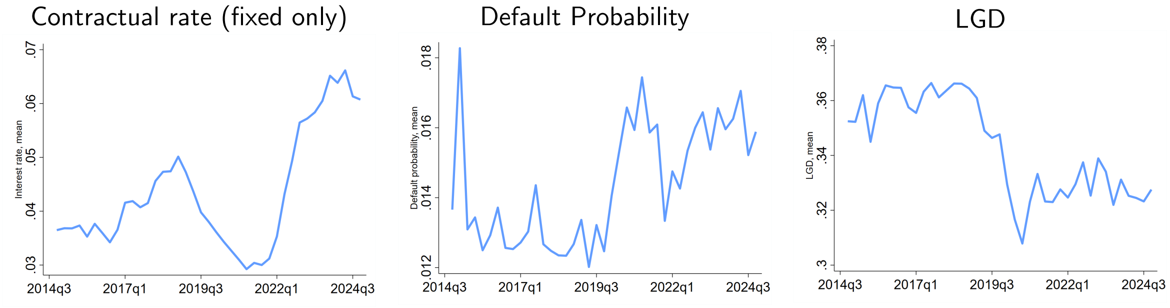
▷ back

Table: Summary Statistics

	mean	sd	p10	p50	p90
Interest rate	4.17	1.69	2.21	3.93	6.59
Maturity (yrs)	6.85	4.64	3.00	5.00	10.25
ρ (%)	3.75	1.69	2.05	3.69	5.88
r^{firm} (%)	2.82	2.75	0.87	3.04	5.26
r^{social} (%)	3.54	1.88	1.77	3.53	5.71
Prob. Default (%)	1.42	2.37	0.19	0.82	2.85
LGD (%)	34.50	13.20	16.00	36.00	50.00
Loan amount (M)	10.77	68.81	1.11	2.55	22.64
Sales (M)	1,254.73	5,923.53	2.17	58.80	1,556.58
Assets (M)	1,770.83	8,956.78	1.06	35.52	1,792.00
Leverage (%)	72.03	24.57	42.57	71.17	100.00
Return on assets (%)	22.61	29.05	4.68	15.56	44.04
N Loans	62687				
N Firms	38587				
N Fixed Rate	31540				
N Variable Rate	31147				

Time series for averages: Contractual Rate, Default, LGD

▷ back



- 2020-2021: Increase in default probability
- Modest decline in losses given default (better recovery)

Data Cleaning and Sample Construction

Sample period: We use FR Y-14Q Schedule H.1 data from 2014Q4 onward **Borrower Filters:**

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

Data Cleaning and Sample Construction

Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only "vanilla" term loans (Facility type = 7)
- Drop loans with:
  - Mixed-rate structures
  - Maturity outside 110 years
  - Implausible interest rates or spreads (outside 1st99th percentile, or $> 50\%$)
  - Missing or invalid PD/LGD values (outside $[0, 1]$)
  - PD $= 1$ (flagged as in default)

Forward interest rate expectations

▷ Back

To estimate ρ for floating rate loans, we need estimates of $\mathbb{E}_0\left[r_t\right]$

- Floating rate loans charge reference rate $+$ spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak et al., 2007)
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out $\mathbb{E}_0\left[r_t\right]$ for each loan, using treasury forward rate plus loan's spread

Lender's discount rate: fixed rate

▷ back

$$1=\sum_{t=1}^{T}\left(\frac{P}{1+\rho}\right)^{t}\left[r+\frac{(1-P)}{P}(1-LGD)\right]+\left(\frac{P}{1+\rho}\right)^{T}$$

Let $x=\frac{P}{1+\rho}$ so

$$1=\left(r+\frac{(1-P)}{P}(1-LGD)\right)\frac{x}{1-x}\left(1-x^{T}\right)+x^{T}$$

Guess that $1+\rho=(1+r)P+(1-P)(1-LGD)$

$$\frac{1-x}{x}=\frac{1}{x}-1=\frac{(1+r)P+(1-P)(1-LGD)}{P}-1=r+\frac{1-P}{P}(1-LGD)$$

And, therefore

$$1=1\left(1-x^{T}\right)+x^{T}$$

which validates the guess.

Firm cost of capital: model

▷ back

$$Q_t = \frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\,Q_{t+1}\right) + \left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{1+\rho}$$

Note that

$$\begin{aligned}
Q_t &= Q_t^P + Q_t^D \\
Q_t^P &= \frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\,Q_{t+1}\right)\right]}{1+\rho} \\
Q_t^D &= \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{1+\rho}
\end{aligned}$$

That is, we strip the bond into the payment in repay $\left(Q_t^P\right)$ and the payment in default $\left(Q_t^D\right)$. Then:

$$\Lambda = \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\,Q_{t+1}\right)\right]} = \frac{Q_t^D}{Q_t^P}$$

Firm cost of capital: measurement

▷ back

The firm defaults with probability $(1-P)$ and the lender recovers $(1-LGD)$. Hence

$$Q_t^{D,data} = \frac{(1-P)(1-LGD)}{1+\rho}$$

For the payment portion notice that at issuance we have the following condition

$$1 = \sum_{s=1}^{T}\left[\frac{P^s \mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}$$

$$1 = \frac{(1-P)(1-LGD)}{1+\rho} + P\frac{\mathbb{E}_t\left[r_{t+1}\right]}{1+\rho} + \left(\sum_{s=2}^{T}\left[\frac{P^s \mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}\right)$$

So, we can define $Q_t^{P,data}$ as $1 = Q_t^{P,data} + Q_t^{D,data}$ so $Q_t^{P,data} = 1 - Q_t^{D,data}$. Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1-P)(1-LGD)}{1+\rho-(1-P)(1-LGD)}$$

Decomposing misallocation

▷ Back

Counterfactual I: What if all lenders have the same $\bar{\rho}$?

$$1 + r^{cf,I}_{social} = \overline{(1+\rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in r^{cf}_{social} → Misallocation due to financial frictions

Counterfactual II: what if we equalize financial frictions?

$$1 + r^{cf,II}_{social} = (1+\rho)\,\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in r^{cf}_{social} → Misallocation due to heterogeneous cost of capital

Variance decomposition

- Decompose total variance in: time, firm, bank, and error
- Keep firms with 5 or more securities

	Time	Bank	Firm	Loan
Contractual rate	71.88	1.63	13.45	13.04
Lender discount rate, ρ	61.94	3.08	14.02	20.96
Firm cost of capital, r^{firm}	33.23	4.25	20.12	42.4
Social cost of capital, r^{social}	53.84	3.87	16.21	26.08
N Firms	1681			
N Securities	14738			

Table: Variance decomposition of interest rates and cost of capital (ρ, r^{firm}, and r^{social})

Estimating $\mathcal{M}$

▷ back

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \times \frac{\partial \log Q}{\partial \log b'}}$$

Given estimates for the function Q, γ, and firm leverage Qb'/k' we can compute $\mathcal{M}$

1. Loans are modeled as perpetuities that decay at a geometric rate θ, we can write Q as the present value of all future payments, discounted at the contractual interest rate r:

$$Q = \frac{\theta + (1-\theta)Q}{1+r} = \frac{\theta}{r+\theta}$$

   r is directly observed in the data, and we can approximate $\theta = 1/T$

2. Guess a functional approximation $Q(z, k, b, \rho)$
3. Estimate $\log \hat{Q}(z, k, b, \rho)$ for every loan origination; compute partial derivatives
4. At steady state, $\gamma = \theta = 1/T$

Estimating $\mathcal{M}$: Q elasticities

▷ back

- We approximate (the log of) Q as a polynomial of investment, borrowing, productivity and ρ
- Investment: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets (Hsieh and Klenow, 2009)
- Approximation:

$$\begin{aligned}\log Q_i &= \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\ &+\beta_{k,k}(\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\ &+\beta_{b,b}(\log b_i)^2 \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\ &+\beta_{z,z}(\log z_i)^2 \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho}(\rho_i)^2 \epsilon_i\end{aligned}$$

- Compute the partial derivatives of $\log Q$ with respect to investment and borrowing.

Estimating $\mathcal{M}$: results

▷ back

- The distribution is extremely concentrated around 1.
- The mean is equal to 0.996 and the median to 0.997, with a standard deviation of 0.006.
- The two measures of misallocation are extremely similar
- Taken together, these results suggest that our assumption that $\mathcal{M} = 1$ is a good one.

Approximating misallocation

▷ back

- Recovery rates from the World Banks Doing Business report
- Approximate r^{social} with ρ in the SS for misallocation
- Use the fixed rate formula for ρ and assume that (P, LGD) are constant across firms
- Approximated cost of misallocation for the US is similar to the actual cost

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