The Cost of Capital and Misallocation in the United States

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Research question and basic idea

Research question: How does dispersion in the cost of capital affect misallocation?

Traditional approach:

- Strong assumptions about production functions (homogeneous Cobb-Douglas)
- $\implies$ Measure heterogeneity in marginal products from cross-sectional data
- $\implies$ Measure misallocation

Our approach:

- Main idea: $r_i + \delta = \text{MPK}_i$
- Combine credit registry data + model to carefully measure cost of capital r_i
- Use heterogeneity in cost of capital to infer cost of misallocation due to imperfect credit markets

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Contribution and findings

Methodological contribution:

- Adapt a standard dynamic corporate finance model to enable measurement using micro data
- Derive a sufficient statistic for misallocation using credit registry data

Empirical Results (US):

- Average cost of capital tracks treasury rates, with a spread
- Measures of cost of capital correlate with traditional measures of $ARPK_i$
- Credit markets seem quite efficient in normal times (losses $\approx 0.9\%$ of GDP)
- Losses from misallocation increased to 1.8% of GDP in 2020-2021

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Related literature

- **Measuring misallocation:**
  - Seminal work: Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
  - Challenge: Standard methods rely on strong assumptions (Haltiwanger et al., 2018).
  - Recent advances: Experimental/quasi-experimental methods to recover marginal products directly (Carrillo et al., 2023; Hughes and Majerovitz, 2025).
  - Contribution: use heterogeneity in funding costs to measure dispersion in MRPK
- **Heterogeneity in the cost of capital:**
  - Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2024)
  - US: Gilchrist, Sim, and Zakrajsek (2013), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
  - Contribution:
    - Estimate firm cost of capital using credit registry data, correcting for loan characteristics, etc.
    - Derive and estimate sufficient statistic for misallocation

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Outline

Model

Welfare and Misallocation

Measurement with credit registry data

Empirical results

ARPK measures and cross-country comparison

Model Summary

- Time discrete and infinite
- Continuum of firms, each matched with a lender
- No aggregate risk (for now - work in progress!)

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- Produce output $f(k_i, z_i)$
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Key question: how do heterogeneity in ρ_i and financial frictions distort the allocation of capital?

Model Equations

Firm value function:

$$V_i\left(k_i, b_i, z_i\right) = \max_{k_i', b_i'} \pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) + \beta \mathbb{E} \overbrace{\left[\max\left\{V_i\left(k_i', b_i', z_i'\right), 0\right\} \middle| z_i\right]}^{\text{Limited liability}}$$

Firm profits:

$$\pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) = f\left(k_i, z_i\right) + (1-\delta) k_i - k_i' - \theta b_i + Q_i\left(k_i', b_i', z_i\right)\left[b_i' - (1-\theta_i) b_i\right]$$

Price of debt:

$$Q_i\left(k_i', b_i', z_i\right) = \frac{\mathbb{E}\left\{\overbrace{\mathcal{P}_i\left(k_i', b_i', z_i'\right)}^{\text{repayment prob.}}\left[\theta_i + (1-\theta_i) Q_i\left(k_i'', b_i'', z_i'\right)\right] + \left(1 - \mathcal{P}_i\left(k_i', b_i', z_i'\right)\right) \overbrace{\frac{\phi_i k_i'}{b_i'}}^{\text{recovery}} \middle| z_i\right\}}{\underbrace{1+\rho_i}_{\text{lender discount rate}}}$$

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Firm optimality

▷ details

- Assume that $\beta < \frac{1}{1+\rho_i}$ so that firm borrows (sufficient, not necessary)

- Combine FOCs for k_i', b_i' as

$$\frac{\mathbb{E}\left[\mathcal{P}_i'(\theta_i + (1-\theta_i)Q_i') \middle| z_i\right]}{Q_i} \times \left[\frac{1 - \frac{\partial Q_i}{\partial k_i'}[b_i' - (1-\theta_i)b_i]}{1 + \frac{\partial Q_i}{\partial b_i'}\frac{[b_i' - (1-\theta_i)b_i]}{Q_i}}\right] = \mathbb{E}\left[\mathcal{P}_i'(f_k(k_i', z_i') + 1 - \delta) \middle| z_i\right]$$

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2. Price impact: summarizes impact of firm's actions on price of debt
3. Expected MRPK

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Firm's cost of capital

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$$1 + r_i^{firm} = \frac{\mathbb{E}\left[\mathcal{P}_i'(\theta_i + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}{Q_i}$$

Lemma 1 (Firm's cost of capital)

The firm's cost of capital is:

$$1 + r_i^{firm} = \frac{1+\rho_i}{1+\Lambda_i} \qquad \Lambda_i := \frac{\mathbb{E}\left[(1-\mathcal{P}_i')\phi_i k_i'/b_i' \middle| k_i', b_i', z_i\right]}{\mathbb{E}\left[\mathcal{P}_i'(\theta + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}$$

▷ *Proof*

- In general, $r_i^{firm} < \rho_i$, since bank recovers something in default, but firm pays zero
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Marginal revenue product of capital (MRPK)

The firm's cost of capital pins down its MRPK

$$\underbrace{(1 + r_i^{firm})\mathcal{M}_i}_{\text{cost of capital}} = \underbrace{\mathbb{E}[\mathcal{P}_i'(f_k(k_i', z_i') + 1 - \delta) |\, k_i', b_i', z_i]}_{\text{expected marginal revenue product of capital}}$$

where $\mathcal{M}_i$ captures the *price impact* of the firm's actions

$$\mathcal{M}_i := \frac{1 - \gamma_i \times \frac{Q_i \cdot b_i'}{k_i'} \times \frac{\partial \log Q_i}{\partial \log k_i'}}{1 + \gamma_i \times \frac{\partial \log Q_i}{\partial \log b_i'}}, \qquad \gamma_i := \frac{b_i' - (1 - \theta_i) b_i}{b_i'}$$

- With low default, $\mathcal{M}_i$ will be very close to 1
- Baseline: set $\mathcal{M}_i = 1$; robustness where we allow for heterogeneous $\mathcal{M}_i$ ▷ Estimate $\mathcal{M}$

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Outline

Aggregate economy and welfare

- Aggregate resource constraint:

$$Y_{t+1} + (1-\delta)K_{t+1} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}_{i,t+1}\left(f(k_{i,t+1}, z_{i,t+1}) + (1-\delta)k_{i,t+1}\right) + (1-\mathcal{P}_{i,t+1}) \cdot \phi_i k_{i,t+1}\right] \mathrm{d}i$$

- Let $\omega_{i,t}(S^t) \in \{0,1\}$ denote whether a firm operates or not
- Assume that existing firms are replaced by identical ones
- Planner's problem:

$$U^* = \max_{\left\{\{k_{i,t}(S^{t-1}), \omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(C_t\right)$$

$$\text{s.t.} \quad \omega_{i,t+1}\left(S^{t+1}\right) \leq \omega_{i,t}\left(S^t\right) \; \forall S^t \subset S^{t+1}, \forall i$$

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Aggregate economy and welfare, cont'd

- Can separate planner's problem into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\left\{K_t, \{\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(\left(\max_{\left\{\{k_{i,t}(S^{t-1})\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} Y_t\right) - I_t\right)$$

- Rewrite inner problem as:

$$Y_t^*\left(K_t, \{\omega_{it}\}_{i\in[0,1]}\right) = \max_{\left\{k_{i,t}^*\right\}_{i\in[0,1]}} \int_0^1 \mathbb{E}_{t-1}\left\{\omega_{it}\cdot f\left(k_{it}^*; z_{it}\right) - \left(1-\omega_{it}\right)\cdot\left[\left(1-\delta\right)k_{it}^* - \phi_i k_{it}^*\right]\right\} di$$

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Aggregate economy and welfare: inner problem

- Redistribute $\{k_{i,t+1}\}_i$ taking exit decisions $\{\mathcal{P}^{DE}_{i,t+1}\}_{i\in[0,1]}$ and K^{DE}_{t+1} as given

$$\max_{\left\{k^*_{i,t+1}\right\}_i} \int_0^1 \mathbb{E}_t\left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^*_{i,t+1}, z_{i,t+1}) + (1-\delta)k^*_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1})\cdot\phi_i k^*_{i,t+1}\right]\mathrm{d}i$$

$$\text{s.t.} \quad \int_0^1 k^*_{i,t+1}\mathrm{d}i = K^{DE}_{t+1}$$

- Lower bound on full misallocation

Social return on capital

- In the decentralized equilibrium:

$$(1 + r_{i,t}^{firm})\mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$

- Define the social marginal product of capital at firm i, $r_{i,t}^{social}(k_{i,t+1})$

$$1 + r_{i,t}^{social}(k_{i,t+1}) \equiv \mathbb{E}\left[\mathcal{P}_{i,t+1}^{DE}\left(f_k\left(k_{i,t+1}, z_{i,t+1}\right) + 1 - \delta\right) + \left(1 - \mathcal{P}_{i,t+1}^{DE}\right)\phi_i\right]$$

social return takes into account recovery in case of default

- Planner Optimality: at $\{k_{i,t+1}^*\}$ the planner **equalizes** $r_{i,t}^{social}(k_{i,t+1}^*)$ across firms
- Equilibrium: dispersion in $r_{i,t}^{social}(k_{i,t+1}^{DE}) \rightarrow$ misallocation

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Misallocation

Proposition 1 (Misallocation)

Misallocation can be measured with $\mathbb{E}\left[r_i^{social}\right]$ *and* $Var\left(r_i^{social}\right)$ *as*

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{Var\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right]+\delta\right)^2}\right)$$

▷ *Proof*

- Extends Hughes and Majerovitz (2025) to a dynamic economy with default
- Measures intensive-margin misallocation only
- Set $\mathcal{E}=\frac{1}{2}$ (elasticity of output w.r.t. $r^{social}+\delta$) and $\delta=0.06$ ▷ Calibration
- **Next:** show how to measure r_i^{social} using credit registry data

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Outline

Model

Welfare and Misallocation

Measurement with credit registry data

Empirical results

ARPK measures and cross-country comparison

Data: FR Y-14Q (Schedule H.1)

▷ cleaning details

- Quarterly loan-level panel on universe of loan facilities $> \$1$M
- Covers top 30/40 BHCs, 2014:Q4-2024Q4
- 91% of C&I undertaken by top 25 banks; 55% of C&I undertaken by all commercial banks
- Detailed information on features of credit facilities
  - Origination date, size, maturity, interest rate/spread, probability of default, loss given default, fixed vs. floating, type of loan, etc.
- Focus on term loans issued to non-government, non-financial US companies

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Summary Statistics

▷ time series

	Mean	St. Dev.	p10	p50	p90
Interest rate	4.18	1.69	2.21	3.94	6.60
Maturity (yrs)	6.83	4.65	3.00	5.00	10.25
Real interest rate	2.39	1.24	0.88	2.33	4.00
Prob. Default (%)	1.45	2.53	0.19	0.85	2.88
LGD (%)	34.41	13.17	16.00	35.60	50.00
Loan amount (M)	10.75	67.58	1.11	2.57	22.92
Sales (M)	1,269.93	6,051.48	2.16	58.50	1,560.10
Assets (M)	1,760.37	8,894.15	1.07	35.55	1,782.22
Leverage (%)	72.17	24.68	42.68	71.29	100.00
Return on assets (%)	27.60	58.51	4.56	15.76	47.81
N Loans	65,284				
N Firms	38,751				
N Fixed Rate	32,592				
N Variable Rate	32,692				

Pricing term loans

For a loan i originated at t, the break-even condition for a lender with discount rate $\rho_{i,t}$ is

$$1 = \sum_{s=1}^{T_{i,t}} \left[\frac{(P_{i,t})^s \cdot \mathbb{E}_t\left(r_{i,t,s}\right) + (P_{i,t})^{s-1} \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}{(1+\rho_{i,t})^s \cdot \mathbb{E}_t(\Pi_{t,s})} \right] + \frac{(P_{i,t})^{T_{i,t}}}{(1+\rho_{i,t})^{T_{i,t}} \cdot \mathbb{E}_t(\Pi_{t,T_{i,t}})}$$

- $T_{i,t}$: maturity
- $P_{i,t}$: repayment probability (constant over time)
- $\mathbb{E}_t[r_{i,t,s}]$: fixed rate or spread over benchmark rate (Gürkaynak et al., 2007) ▷ forward rates
- $LGD_{i,t}$: loss given default (constant over time)
- $\mathbb{E}_t(\Pi_{t,s})$: total expected inflation from t to s (Cleveland Fed)
- $\Rightarrow$ Solve for lender's discount rate: $\rho_{i,t}$ ▷ fixed real rate

Measuring Firm and Social Cost of Capital

Lemma 2 (Firm cost of capital)

We can write the firm cost of capital as

$$1 + r_{i,t}^{firm} = (1 + \rho_{i,t}) - (1 - P_{i,t})(1 - LGD_{i,t})$$

▷ *Proof*

Lemma 3 (Social cost of capital)

The social cost of capital can be written as:

$$\begin{aligned} 1 + r_{i,t}^{social} &= (1 + r_{i,t}^{firm})\mathcal{M}_{i,t} + (1 - P_{i,t})(1 - LGD_{i,t}) lev_{i,t} \\ &= \underbrace{(1 + \rho_{i,t})\,\mathcal{M}_{i,t}}_{\text{lender discount rate}} + \underbrace{(lev_{i,t} - \mathcal{M}_{i,t}) \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}_{\text{wedge due to financial frictions}} \end{aligned}$$

In general, for $lev_{i,t} \in (0, 1)$, we have that $r^{firm} \leq r^{social} \leq \rho$

Sufficient statistic for misallocation

$$\log\left(Y_t^*/Y_t^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{\text{Var}\left(r_{i,t}^{social}\right)}{\left(\mathbb{E}\left[r_{i,t}^{social}\right]+\delta\right)^2}\right)$$

$$1+r_{i,t}^{social} = (1+\rho_{i,t})\,\mathcal{M}_{i,t} + (lev_{i,t}-\mathcal{M}_{i,t})\cdot(1-P_{i,t})\cdot(1-LGD_{i,t})$$

- Set $\mathcal{M}_{i,t}=1$; reasonable approximation given our model

▷ Estimate $\mathcal{M}$

- Can measure misallocation directly with credit registry data!
- Dispersion in $r_{i,t}^{social}$ comes from:
  1. Dispersion in lender's discount rate, $\rho_{i,t}$
  2. Dispersion in financial frictions wedge
  3. Covariance between $\rho_{i,t}$ and financial frictions wedge

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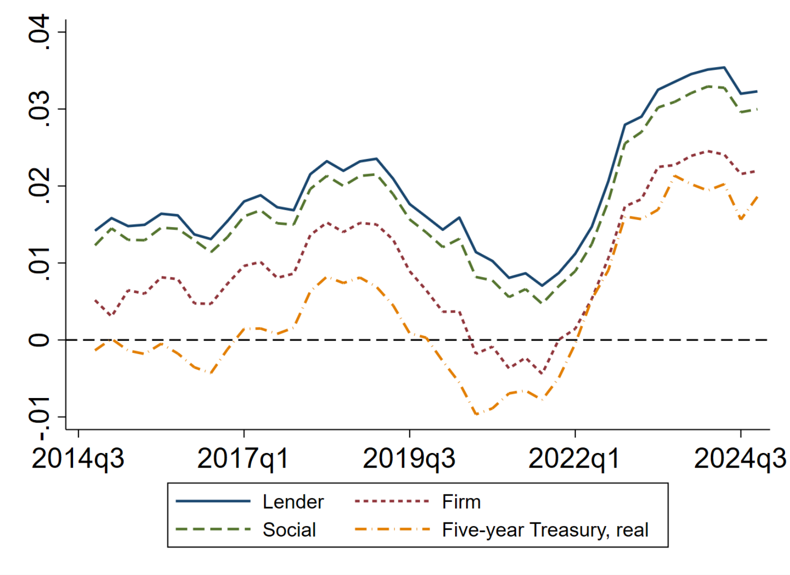
Estimates for lender discount rate, firm and social cost of capital

	Mean	SD	p10	p50	p90
ρ (%)	1.87	1.55	0.41	1.88	3.62
r^{firm} (%)	0.92	2.80	-0.86	1.26	3.03
r^{social} (%)	1.66	1.78	0.12	1.73	3.47

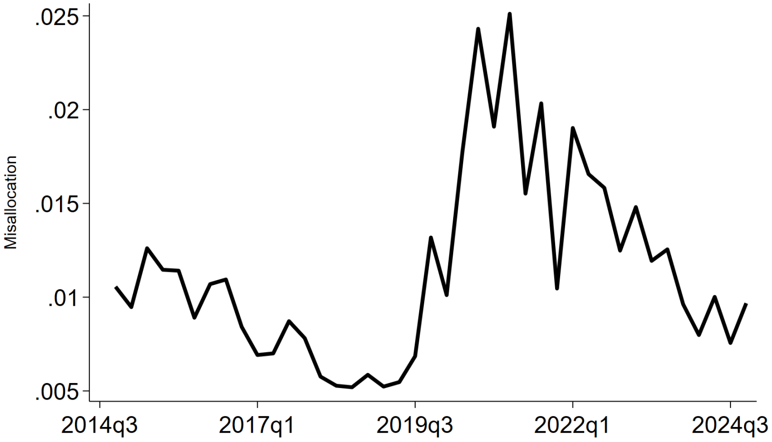
- Financial frictions/recovery: $\mathbb{E}\left[r_{i,t}^{firm}\right] < \mathbb{E}\left[r_{i,t}^{social}\right], \mathbb{E}\left[\rho_{i,t}\right]$
- ▷ Variance decomposition

Time series for average discount rate, firm and social cost of capital



Misallocation in the US, 2014-2024

▷ heterogeneous $\mathcal{M}$



- About 0.9% before 2020
- ↑ to 1.8% in 2020-2021
- ↓ to 1.2% in 2022-2024

The 2020–2021 increase in misallocation

1. Predominantly explained by changes in dispersion in ρ_i, rather than financial frictions ▷ details

2. Sharp rise in the coefficient of variation of ρ_i ▷ details

3. ρ_i dispersion $\uparrow$ due to increased dispersion of expected losses ▷ details

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r^{social} correlates with standard measures of ARPK

	(1)	(2)	(3)	(4)	(5)
	$\log(ARPK)$, Sales	$\log(ARPK)$, EBITDA	$\log(ARPK)$, Sales	$\log(ARPK)$, EBITDA	$\log(ARPK)$, VA
$\log(r^{social} + \delta)$	0.15***	0.24***	0.16**	0.15*	0.39***
	(0.03)	(0.04)	(0.07)	(0.08)	(0.07)
Observations	59294	57334	4184	4072	3432
Adj. R2	0.27	0.22	0.68	0.52	0.61
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

ARPK-based misallocation

▷ other measures

Focus on Compustat firms to make measures comparable

	$r^{social} + \delta$	$\frac{\text{Sales}}{\text{Capital}}$	$\frac{\text{EBITDA}}{\text{Capital}}$	$\frac{\text{Value Added}}{\text{Capital}}$
$Var(\log)$	0.01	0.19	0.24	0.21
Misallocation (%)	0.36	4.75	6.20	5.28

- Our measure looks only at misallocation coming from heterogeneity in the cost of capital
- ...but does not require detailed data on firm financials (i.e., value added)
- $\implies$ directly applicable to most existing credit registries

Cross-country comparison, approximation

▷ details

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
Mean real rate, %	66.8	8.00	83.0	12.4	1.4
SD real rate, %	38.1	2.9	93.3	5.2	1.2
Mean def. prob., %	2.7	16.9	4.0	8.9	1.5
Mean recovery rate, %	42.8	42.8	18.2	63.9	66.6
Implied misallocation, %	6.5	13.5	21.5	2.8	0.8

- Developing countries: higher mean and standard deviation of real interest rates
- U.S.: lower mean and standard deviation of interest rates, higher recovery
- Brazil: most extreme misallocation: 21.5%.

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Conclusion

- Develop a framework to measure misallocation using credit registry data
  1. Standard dynamic corporate finance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Relies on standard credit registry variables as inputs (r, P, LGD, T, etc.)
- Application to U.S. credit registry data (FR Y-14Q)
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Appendices

Firm FOC: details

▷ back

Firm FOCs:

$$[k_i'] : \quad -1 + \frac{\partial Q_i(k_i', b_i', z_i)}{\partial k_i'}[b_i' - (1-\theta_i)b_i] + \beta\mathbb{E}\left\{\mathcal{P}_i(k_i', b_i', z_i')[f_k(k_i', z_i') + 1 - \delta] \middle| z_i\right\} = 0$$

$$[b_i'] : \quad \frac{\partial Q_i(k_i', b_i', z_i)}{\partial b_i'}[b_i' - (1-\theta_i)b_i] + Q_i(k_i', b_i', z_i) - \beta\mathbb{E}\left\{\mathcal{P}_i(k_i', b_i', z_i')[\theta_i + (1-\theta_i)Q_i(k_i'', b_i'', z_i')] \middle| z_i\right\}$$

$$= 0$$

Firm's cost of capital: proof

▷ back

$$\frac{1}{Q_t}\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right] = \frac{(1+\rho)\,\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]+\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}$$

$$= (1+\rho)\left(1+\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}\right)^{-1}$$

$$= (1+\rho)\,(1+\Lambda)^{-1}$$

where

$$\Lambda \equiv \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}$$

Misallocation formula: proof

▷ back

- Formally, planner's problem is now the same as solving $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i)\mathrm{d}i$, where $f_i(k_i)$ is now expected output

- Apply Hughes and Majerovitz (2024), noting $\frac{dY}{dk} = r^{social} + \delta$

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right] + \delta\right)^2}\right)$$

- $\mathcal{E}$ is (negative) elasticity of output w.r.t. cost of capital $(r^{social} + \delta)$

Calibration of $\mathcal{E}$

▷ back

- $\mathcal{E}_i$ is the elasticity of expected output with respect to the cost of capital
- Assume that $f(k,z) = z \cdot k^{\alpha}$ and there is no default, then

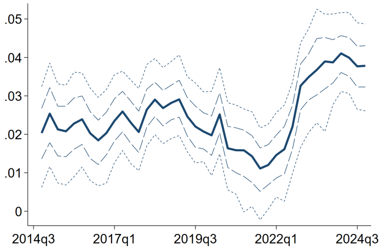
$$\mathcal{E} = \frac{\alpha}{1-\alpha}$$

- $\alpha = \frac{1}{3}$ implies $\mathcal{E} = \frac{1}{2}$

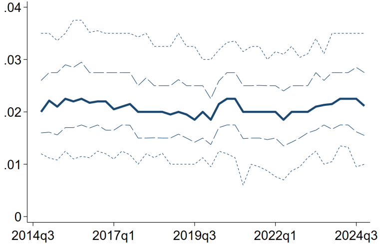
Time series for averages: real interest rate, PD, LGD

▷ back

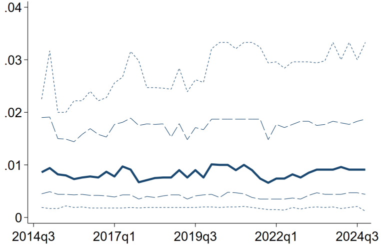
Real interest rate



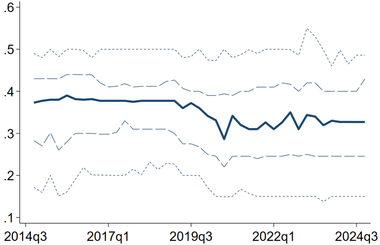
Interest rate spread (var.)



Probability of default



Loss given default



Data cleaning and sample construction

▷ back

We use FR Y-14Q Schedule H.1 data from 2014Q4 to 2024Q4.

Borrower Filters:

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

Data cleaning and sample construction, cont'd

▷ back

Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only "vanilla" term loans (Facility type $= 7$)
- Drop loans with:
  - Mixed-interest rate structures
  - Maturity less than 1 year or longer than 10 years
  - Implausible interest rates or spreads (outside 1st - 99th percentile)
  - Missing or invalid PD/LGD values (outside $[0, 1]$)
  - PD $= 1$ (flagged as in default)

Forward interest rate expectations

▷ back

To estimate ρ_i for floating rate loans, need estimates of $\mathbb{E}_0[r_t] + s_i$

- Floating rate loans charge reference rate + spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak et al., 2007)
- Average spread between SOFR and Treasury rates 2018-2025 $\simeq$ 2 basis points
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out $\mathbb{E}_0[r_t] + s_i$ for each loan, using treasury forward rate plus loan's spread

Firm cost of capital: model

▷ back

$$Q_t = \frac{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta)\, Q_{t+1}\right) + \left(1 - \mathcal{P}_{t+1}\right) \phi k_{t+1}/b_{t+1}\right]}{1+\rho}$$

Note that

$$\begin{aligned} Q_t &= Q_t^P + Q_t^D \\ Q_t^P &= \frac{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta)\, Q_{t+1}\right)\right]}{1+\rho} \\ Q_t^D &= \frac{\mathbb{E}_t \left[\left(1 - \mathcal{P}_{t+1}\right) \phi k_{t+1}/b_{t+1}\right]}{1+\rho} \end{aligned}$$

That is, we strip the bond into the payment in repay $\left(Q_t^P\right)$ and the payment in default $\left(Q_t^D\right)$. Then:

$$\Lambda = \frac{\mathbb{E}_t \left[\left(1 - \mathcal{P}_{t+1}\right) \phi k_{t+1}/b_{t+1}\right]}{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta)\, Q_{t+1}\right)\right]} = \frac{Q_t^D}{Q_t^P}$$

Firm cost of capital: measurement

▷ back

The firm defaults with probability $(1-P)$ and the lender recovers $(1-LGD)$. Hence

$$Q_t^{D,data} = \frac{(1-P)(1-LGD)}{1+\rho}$$

For the payment portion notice that at issuance we have the following condition

$$1 = \sum_{s=1}^{T}\left[\frac{P^s \mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}$$

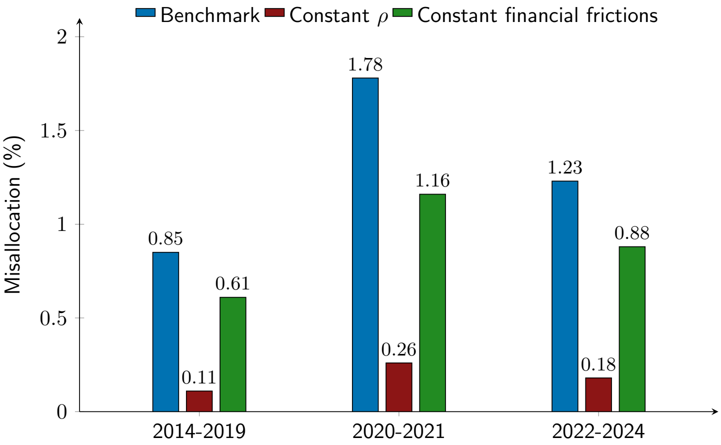
$$1 = \frac{(1-P)(1-LGD)}{1+\rho} + P\frac{\mathbb{E}_t\left[r_{t+1}\right]}{1+\rho} + \left(\sum_{s=2}^{T}\left[\frac{P^s \mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}\right)$$

So, we can define $Q_t^{P,data}$ as $1 = Q_t^{P,data} + Q_t^{D,data}$ so $Q_t^{P,data} = 1 - Q_t^{D,data}$. Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1-P)(1-LGD)}{1+\rho-(1-P)(1-LGD)}$$

1. The 2020-21 rise in misallocation was driven by $\{\rho_i\}$

▷ details ▷ back



Decomposing misallocation

▷ back

Counterfactual I: What if all lenders have the same $\bar{\rho}$?

$$1 + r^{cf,I}_{social} = \overline{(1+\rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in r^{cf}_{social} $\rightarrow$ Misallocation due to financial frictions

Counterfactual II: what if we equalize financial frictions?

$$1 + r^{cf,II}_{social} = (1+\rho)\,\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in r^{cf}_{social} $\rightarrow$ Misallocation due to heterogeneous cost of capital

2. The CV of ρ_i increased during 2020-21

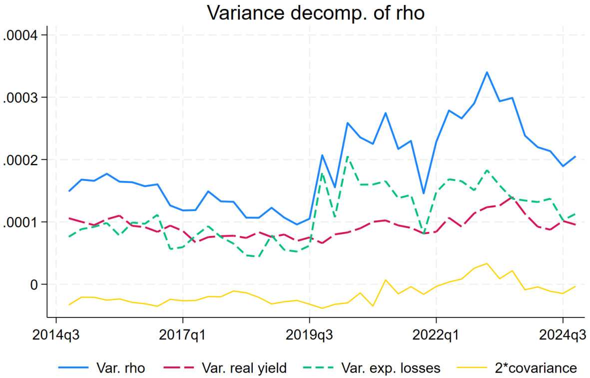
▷ back

- As policy rates decreased in 2020-21, so did mean ρ_i
- Standard deviation of ρ_i increased during this period

3. Variance of ρ related to variance of expected losses

▷ details ▷ back

$$\rho_i = \underbrace{\rho_i(P_i = 1)}_{\text{real yield}} + \underbrace{[\rho_i - \rho_i(P_i = 1)]}_{\text{exp. losses}}$$



- $\sigma(\rho) \uparrow$ due to $\sigma(\text{exp. losses}) \uparrow$
- $\sigma(\text{exp. losses}) \uparrow$ without $\sigma(\text{yield}) \uparrow$
- Possibly tied to underpricing of risky loans, implicit guarantees, etc.

Decomposing ρ

▷ back

- The "real yield" is the implied $\rho^*_{i,t}$ when $P_{i,t} = 1$

$$1 = \sum_{s=1}^{T_{i,t}} \left[\frac{\mathbb{E}_t\left(r_{i,t,s}\right)}{\left(1+\rho^*_{i,t}\right)^s \cdot \mathbb{E}_t(\Pi_{t,s})} \right] + \frac{1}{\left(1+\rho^*_{i,t}\right)^{T_{i,t}} \cdot \mathbb{E}_t(\Pi_{t,T_{i,t}})}$$

- Real yield independent of $P_{i,t}, LGD_{i,t}$

- Only affected by losses through the contractual rate r

Variance decomposition

▷ back

- Decompose total variance in: time, firm, bank, and error
- Keep firms with 5 or more securities

	Time	Bank	Firm	Loan
Contractual rate	69.08	1.68	14.72	14.52
Real rate	49.35	3.62	25.32	21.71
ρ	43.07	3.61	22.93	30.39
r^{firm}	16.5	3.73	30.88	48.9
r^{social}	34.72	4.21	24.94	36.13
N Firms	1844			
N Loans	16088			

Table: Variance decomposition of interest rates and cost of capital (ρ, r^{firm}, and r^{social})

Estimating $\mathcal{M}$

▷ back

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \times \frac{\partial \log Q}{\partial \log b'}}$$

Need Q, γ, and firm leverage Qb'/k' to compute $\mathcal{M}$

1. To compute Q, assume that loans are perpetuities that decay at a geometric rate θ, discounted at the loan's real interest rate r:

$$Q = \frac{\theta + (1-\theta)Q}{1+r} = \frac{\theta}{r+\theta}$$

   r is directly observed in the data, and we can approximate $\theta = 1/T$

2. Guess a functional approximation $Q(z, k, b, \rho)$
3. Estimate $\log \hat{Q}(z, k, b, \rho)$ for every loan origination; compute partial derivatives
4. At steady state, $\gamma = \theta = 1/T$

Estimating $\mathcal{M}$: Q elasticities

▷ back

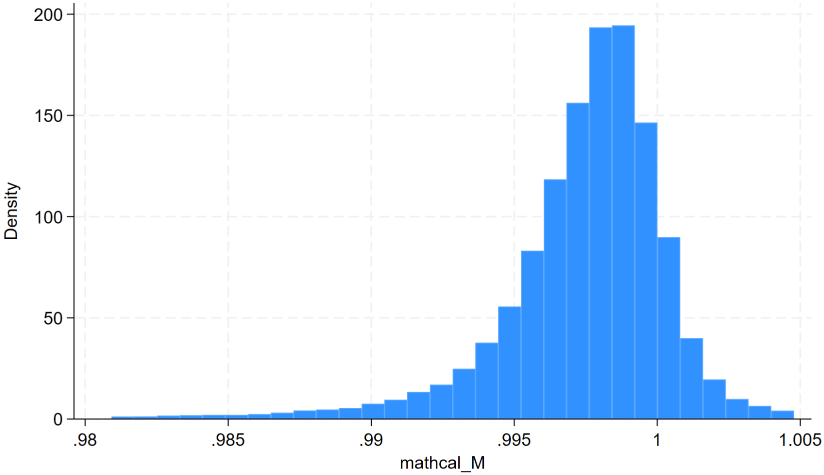
- We approximate (the log of) Q as a polynomial of firm capital, borrowing, productivity and ρ

$$
\begin{aligned}
\log Q_i = \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\
+\beta_{k,k}(\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\
+\beta_{b,b}(\log b_i)^2 + \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\
+\beta_{z,z}(\log z_i)^2 + \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho}(\rho_i)^2 + \epsilon_i
\end{aligned}
$$

- Capital: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets
- This allows us to compute $\frac{\partial \log Q}{\partial \log k'}$ and $\frac{\partial \log Q}{\partial \log b'}$

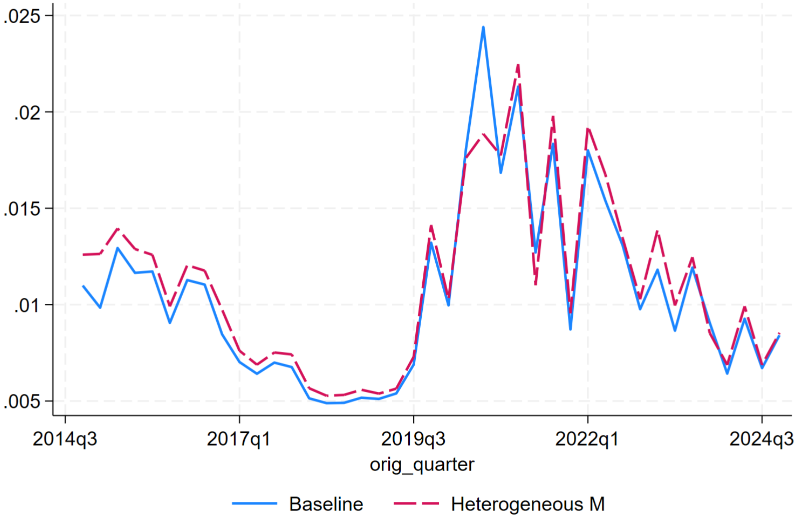
Estimating $\mathcal{M}$: results

▷ back



Misallocation with heterogeneous $\mathcal{M}$

▷ back



ARPK-based misallocation measures

▷ back

	(1) log $ARPK$, Sales	(2) log $ARPK$, EBITDA	(3) log $ARPK$, Sales	(4) log $ARPK$, EBITDA	(5) log $ARPK$, VA
$\log(r^{social} + \delta)$	0.15***	0.24***	0.16**	0.15*	0.39***
	(0.03)	(0.04)	(0.07)	(0.08)	(0.07)
Observations	59294	57334	4184	4072	3432
Adj. R2	0.27	0.22	0.68	0.52	0.61
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat
Var($\log ARPK$)	1.97	1.52	0.19	0.24	0.21
Misalloc., $ARPK$, %	63.63	46.08	4.75	6.20	5.28
Var($\log(r^{social} + \delta)$)	0.04	0.04	0.01	0.01	0.01
Misalloc., $r^{social} + \delta$, %	0.96	0.96	0.36	0.36	0.36

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Details on cross-country comparison

▷ back to loan pricing ▷ back to cross-country

- For a fixed real interest rate $r_{i,t}$, ρ has a closed-form:

$$1 + \rho_{i,t} = P_{i,t}\left(1 + r_{i,t}\right) + \left(1 - P_{i,t}\right)\left(1 - LGD_{i,t}\right)$$

- Assume all loans have the same maturity:
  1. Obtain mean real rate by subtracting average realized inflation from mean nominal rate
  2. Inflation should not affect standard deviation of nominal rates (or spreads)
- Assume all loans have the same $P_{i,t}, LGD_{i,t}$, equal to the average
- Recovery rates and inflation rates from the World Bank
- Approximate $r_{i,t}^{social} \simeq \rho_{i,t}$ and compute misallocation using our formula:

$$\log(Y_t^*/Y_t^{DE}) = \frac{1}{2}\mathcal{E}\log\left(1 + \frac{Var(\rho_{i,t})}{(\mathbb{E}[\rho_{i,t}] + \delta)^2}\right)$$