The Cost of Capital and Misallocation in the United States

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Misallocation and the cost of capital

Research question: How does dispersion in the cost of capital affect misallocation?

Traditional approach:

1. Strong assumptions about production functions
2. Measure heterogeneity in marginal products from cross-sectional input/balance sheet data
3. Estimate capital misallocation

Our approach:

- Optimizing firms equate cost of capital to expected marginal product of capital
- Combine credit registry data + model to carefully measure cost of capital, and infer MPK
- Use dispersion in cost of capital to quantify welfare losses stemming from credit market frictions

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Methodological contribution:

- Adapt a standard dynamic corporate finance model for measurement with micro loan-level data
- Derive a sufficient statistic for capital misallocation arising due to credit market frictions

Empirical results for the US:

- Average cost of capital tracks 5-year treasury rates, with a spread
- Measures of cost of capital correlate with traditional measures of $ARPK_i$ at the firm level
- Credit markets efficient in normal times: losses from misallocation $\approx 0.9\%$ of GDP
- Losses from misallocation increased to 1.8% of GDP in 2020-2021

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Related literature

- **Measuring misallocation:**
  - Seminal work: Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
  - Challenge: Standard methods rely on strong assumptions (Haltiwanger et al., 2018).
  - Recent advances: quasi-experimental methods to recover marginal products directly (Carrillo et al., 2024; Hughes and Majerovitz, 2025).
  - Contribution: use heterogeneity in funding costs to measure dispersion in MRPK
- **Heterogeneity in the cost of capital:**
  - Developing countries:Banerjee and Duflo (2005); Cavalcanti, Kaboski, Martins, and Santos (2024)
  - US: Gilchrist, Sim, and Zakrajsek (2013); David, Schmid, and Zeke (2022); Gormsen and Huber (2023, 2024); Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
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Outline

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- Discrete time, infinite horizon
- Continuum of firms, each matched with a lender
- No aggregate risk (for now - work in progress!)

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Key question: how do heterogeneity in ρ_i and financial frictions distort the allocation of capital?

Model

Firm value function:

$$V_i(k_i, b_i, z_i) = \max_{k_i', b_i'} \pi_i(k_i, b_i, z_i, k_i', b_i') + \beta \mathbb{E} \overbrace{\left[\max\{V_i(k_i', b_i', z_i'), 0\} \middle| z_i \right]}^{\text{Limited liability}}$$

Firm profits:

$$\pi_i(k_i, b_i, z_i, k_i', b_i') = f(k_i, z_i) + (1-\delta) k_i - k_i' - \theta b_i + Q_i(k_i', b_i', z_i)\left[b_i' - (1-\theta_i) b_i\right]$$

Price of debt:

$$Q_i(k_i', b_i', z_i) = \frac{\mathbb{E}\left\{ \overbrace{\mathcal{P}_i(k_i', b_i', z_i')}^{\text{repayment prob.}} \left[\theta_i + (1-\theta_i) Q_i(k_i'', b_i'', z_i')\right] + (1 - \mathcal{P}_i(k_i', b_i', z_i')) \overbrace{\frac{\phi_i k_i'}{b_i'}}^{\text{recovery}} \middle| z_i \right\}}{\underbrace{1+\rho_i}_{\text{lender discount rate}}}$$

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Firm optimality

▷ details

- **Cost of capital:**

$$\underbrace{\frac{\mathbb{E}\left[\mathcal{P}_i'(\theta_i + (1-\theta_i)Q_i') \middle| z_i\right]}{Q_i}}_{1+r_i^{\text{firm}}} \times \underbrace{\left[\frac{1 - \frac{\partial Q_i}{\partial k_i'}[b_i' - (1-\theta_i)b_i]}{1 + \frac{\partial Q_i}{\partial b_i'}\frac{[b_i' - (1-\theta_i)b_i]}{Q_i}}\right]}_{\mathcal{M}_i}$$

- $1 + r_i^{\text{firm}}$: implied interest rate perceived by the firm
- $\mathcal{M}_i$: price impact term capturing how (k_i', b_i') affect debt price Q_i
- **Optimality:** firm equates cost of capital to expected MRPK

$$(1 + r_i^{\text{firm}}) \cdot \mathcal{M}_i = \underbrace{\mathbb{E}\left[\mathcal{P}_i'\left(f_k(k_i', z_i') + 1 - \delta\right) \middle| z_i\right]}_{\text{expected MRPK}}$$

- Measurement idea: measure r_i^{firm} from loan data to infer dispersion in MRPK and misallocation.

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Firm's cost of capital

Lemma 1 (Firm's cost of capital)

The firm's cost of capital is:

$$1 + r_i^{firm} = \frac{1+\rho_i}{1+\Lambda_i} \qquad \Lambda_i := \frac{\mathbb{E}\left[(1-\mathcal{P}_i')\,\phi_i k_i'/b_i' \mid k_i', b_i', z_i\right]}{\mathbb{E}\left[\mathcal{P}_i'\left(\theta + (1-\theta_i)Q_i'\right) \mid k_i', b_i', z_i\right]}$$

▷ *Proof*

- Financial frictions wedge: $\Lambda_i > 0$, if expected recovery is positive
- $r_i^{firm} < \rho_i$, since lender recovers something in default, but firm pays zero in those states

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Outline

1. Model
2. **Welfare and misallocation**
3. Measurement with credit registry data
4. Empirical results
5. Extensions & robustness

Aggregate economy and welfare

- Aggregate resources available for consumption and new capital:

$$Y_{t+1} + (1-\delta)K_{t+1} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}_{i,t+1}\left(f(k_{i,t+1}, z_{i,t+1}) + (1-\delta)k_{i,t+1}\right) + (1-\mathcal{P}_{i,t+1}) \cdot \phi_i k_{i,t+1}\right] \mathrm{d}i$$

- Let $\omega_{i,t}(S^t) \in \{0,1\}$ denote whether a firm operates or not
- Assume that existing firms are replaced by identical ones
- Planner's problem:

$$U^* = \max_{\left\{\{k_{i,t}(S^{t-1}),\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u(C_t)$$

$$\text{s.t.} \qquad K_t = \int_0^1 k_{i,t}(S^{t-1})\mathrm{d}i$$

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t$$

$$\omega_{i,t+1}\left(S^{t+1}\right) \leq \omega_{i,t}\left(S^t\right) \; \forall S^t \subset S^{t+1}, \forall i$$

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Aggregate economy and welfare

- Can separate planner's problem into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\left\{K_t, \{\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(\left(\max_{\left\{\{k_{i,t}(S^{t-1})\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} Y_t\right) - I_t\right)$$

- Write inner problem as:

$$Y_t^*\left(K_t, \{\omega_{it}\}_{i\in[0,1]}\right) = \max_{\left\{k_{i,t}^*\right\}_{i\in[0,1]}} \int_0^1 \mathbb{E}_{t-1}\left\{\omega_{it} \cdot f\left(k_{it}^*; z_{it}\right) - \left(1-\omega_{it}\right) \cdot \left[\left(1-\delta\right) k_{it}^* - \phi_i k_{it}^*\right]\right\} di$$

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Aggregate economy and welfare: intensive-margin misallocation

- Inner problem: reallocate capital across firms, taking exit and aggregate capital as given

- Focus on misallocation at the intensive margin
  - As in most of the literature: Restuccia and Rogerson (2008); Hsieh and Klenow (2009)
  - Necessary for measurement: we do not observe loans that are never originated

- Planner redistributes $\{k_{i,t+1}\}_{i\in[0,1]}$ taking exit decisions $\{\mathcal{P}^{DE}_{i,t+1}\}_{i\in[0,1]}$ and K^{DE}_{t+1} as given

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Social return on capital

- Define the social marginal product of capital at firm i, $r_{i,t}^{social}(k)$, as:

$$1 + r_{i,t}^{social}(k) \equiv \mathbb{E}\left[\mathcal{P}_{i,t+1}^{DE}\left(f_k\left(k, z_{i,t+1}\right) + 1 - \delta\right) + \left(1 - \mathcal{P}_{i,t+1}^{DE}\right)\phi_i\right]$$

social return takes into account recovery in case of default

- Planner optimality: at $\{k_{i,t+1}^*\}_i$ the planner **equalizes** $r_{i,t}^{social}(k_{i,t+1}^*)$ across firms
- In the decentralized equilibrium:

$$(1 + r_{i,t}^{firm})\mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$
$$\Rightarrow 1 + r_{i,t}^{social}(k_{i,t+1}^{DE}) = (1 + r_{i,t}^{firm})\mathcal{M}_{i,t} + (1 - \mathcal{P}_{i,t+1}^{DE})\phi_i$$

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$$1 + r_{i,t}^{social}(k) \equiv \mathbb{E}\left[\mathcal{P}_{i,t+1}^{DE}\left(f_k\left(k, z_{i,t+1}\right) + 1 - \delta\right) + \left(1 - \mathcal{P}_{i,t+1}^{DE}\right)\phi_i\right]$$

social return takes into account recovery in case of default

- Planner optimality: at $\{k_{i,t+1}^{*}\}_i$ the planner **equalizes** $r_{i,t}^{social}(k_{i,t+1}^{*})$ across firms

- In the decentralized equilibrium:

$$\begin{aligned}(1 + r_{i,t}^{firm})\mathcal{M}_{i,t} = \; & \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)] \\ \Rightarrow 1 + r_{i,t}^{social}(k_{i,t+1}^{DE}) = & \; (1 + r_{i,t}^{firm})\mathcal{M}_{i,t} + (1 - \mathcal{P}_{i,t+1}^{DE})\phi_i\end{aligned}$$

- Equilibrium: dispersion in $r_{i,t}^{social}(k_{i,t+1}^{DE}) \rightarrow$ misallocation

Misallocation: sufficient statistic

Proposition 1 (Misallocation)

Misallocation can be measured with $\mathbb{E}\left[r_i^{social}\right]$ *and* $Var\left(r_i^{social}\right)$ *as*

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{Var\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right] + \delta\right)^2}\right)$$

▷ *Proof*

- Extends Hughes and Majerovitz (2025) to a dynamic economy with default
  - Measures intensive-margin misallocation
  - 2nd order approx. for arbitrary f_i; exact when production is CD, and (z, μ) are jointly $\log \mathcal{N}$
  - Set $\mathcal{E} = \frac{1}{2}$ (elasticity of output w.r.t. $r^{social} + \delta$) and $\delta = 0.06$ ▷ calibration
- **Next:** show how to measure r_i^{social} using credit registry data

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Outline

1. Model
2. Welfare and misallocation
3. **Measurement with credit registry data**
4. Empirical results
5. Extensions & robustness

Data: FR Y-14Q (Schedule H.1)

▷ sample details

- Quarterly loan-level panel on universe of loan facilities $> \$1$M
- Sample covers top 40 BHCs, 2014:Q4-2024:Q4 ($\simeq 91\%$ of C&I lending)
- Detailed information on features of credit facilities
  - Origination date, size, maturity, interest rate/spread, probability of default, loss given default, fixed vs. floating, type of loan, etc.
- Focus on term loans at origination, issued to non-government, non-financial US companies

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Summary statistics

▷ time series

	Mean	St. Dev.	p10	p50	p90
Interest rate	4.18	1.69	2.21	3.94	6.60
Maturity (yrs)	6.83	4.65	3.00	5.00	10.25
Real interest rate	2.39	1.24	0.88	2.33	4.00
Prob. Default (%)	1.45	2.53	0.19	0.85	2.88
LGD (%)	34.41	13.17	16.00	35.60	50.00
Loan amount (M)	10.75	67.58	1.11	2.57	22.92
Sales (M)	1,269.93	6,051.48	2.16	58.50	1,560.10
Assets (M)	1,760.37	8,894.15	1.07	35.55	1,782.22
Leverage (%)	72.17	24.68	42.68	71.29	100.00
Return on assets (%)	27.60	58.51	4.56	15.76	47.81
N Loans	65,284				
N Firms	38,751				
N Fixed Rate	32,592				
N Variable Rate	32,692				

Pricing term loans

For a loan i originated at t, the break-even condition for a lender with discount rate $\rho_{i,t}$ is

$$1 = \sum_{s=1}^{T_{i,t}} \left[\frac{(P_{i,t})^s \cdot \mathbb{E}_t\left(r_{i,t,s}\right) + (P_{i,t})^{s-1} \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}{(1+\rho_{i,t})^s \cdot \mathbb{E}_t(\Pi_{t,s})} \right] + \frac{(P_{i,t})^{T_{i,t}}}{(1+\rho_{i,t})^{T_{i,t}} \cdot \mathbb{E}_t(\Pi_{t,T_{i,t}})}$$

- $T_{i,t}$: maturity
- $P_{i,t}$: repayment probability (constant over time)
- $LGD_{i,t}$: loss given default (constant over time)
- $\mathbb{E}_t[r_{i,t,s}]$: fixed rate or spread over benchmark rate (Gürkaynak, Sack, and Wright, 2007) ▷ forward rates
- $\mathbb{E}_t(\Pi_{t,s})$: total expected inflation between t and s, from term structure of $\mathbb{E}_t\pi_s$ (Cleveland Fed)
- $\Rightarrow$ Solve for lender's discount rate: $\rho_{i,t}$ ▷ fixed real rate

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Measuring firm and social cost of capital

Lemma 2 (Firm and social cost of capital)

We can write the firm cost of capital as:

$$1 + r_{i,t}^{firm} = (1 + \rho_{i,t}) - (1 - P_{i,t})(1 - LGD_{i,t})$$

and the social cost of capital as:

$$\begin{aligned} 1 + r_{i,t}^{social} &= (1 + r_{i,t}^{firm})\mathcal{M}_{i,t} + (1 - P_{i,t})(1 - LGD_{i,t})lev_{i,t} \\ &= \underbrace{(1 + \rho_{i,t})\,\mathcal{M}_{i,t}}_{\textit{lender discount rate}} + \underbrace{(lev_{i,t} - \mathcal{M}_{i,t}) \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}_{\textit{wedge due to financial frictions}} \end{aligned}$$

▷ *Proof*

In general, $r^{firm} \leq r^{social} \leq \rho$: firms perceive lower cost of capital than lenders, due to recovery

Sufficient statistic for misallocation

$$\log\left(Y_t^*/Y_t^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{\mathrm{Var}\left(r_{i,t}^{social}\right)}{\left(\mathbb{E}\left[r_{i,t}^{social}\right]+\delta\right)^2}\right) \quad (1)$$

$$1+r_{i,t}^{social} = (1+\rho_{i,t})\,\mathcal{M}_{i,t} + (lev_{i,t}-\mathcal{M}_{i,t})\cdot(1-P_{i,t})\cdot(1-LGD_{i,t}) \quad (2)$$

- Set $\mathcal{M}_{i,t}=1$; reasonable approximation given our data ▷ estimate $\mathcal{M}$
- Can measure misallocation directly with credit registry data using (1) and (2)!
- Dispersion in $r_{i,t}^{social}$ comes from:
  1. Dispersion in lender's discount rate, $\rho_{i,t}$
  2. Dispersion in financial frictions wedge
  3. Covariance between $\rho_{i,t}$ and financial frictions wedge

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Outline

1. Model
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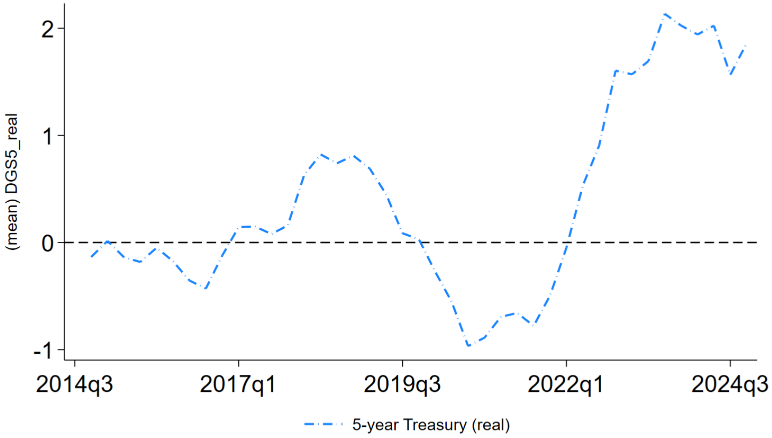
Time series for average discount rate, firm and social cost of capital



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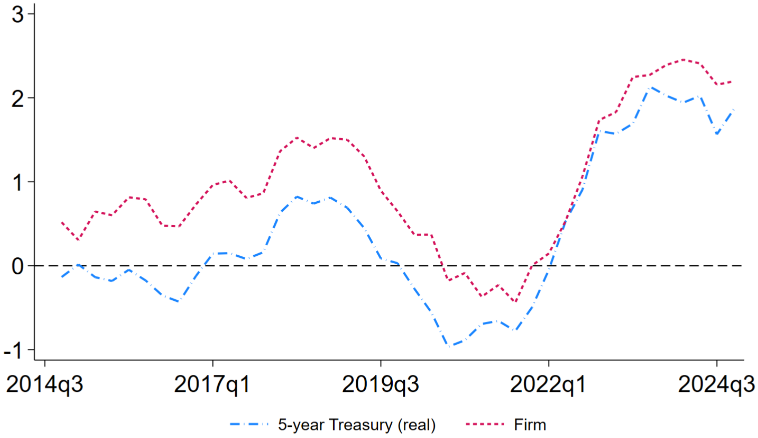

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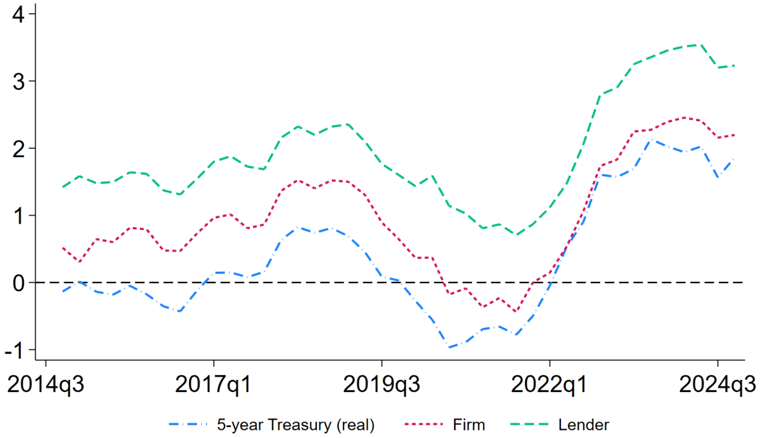

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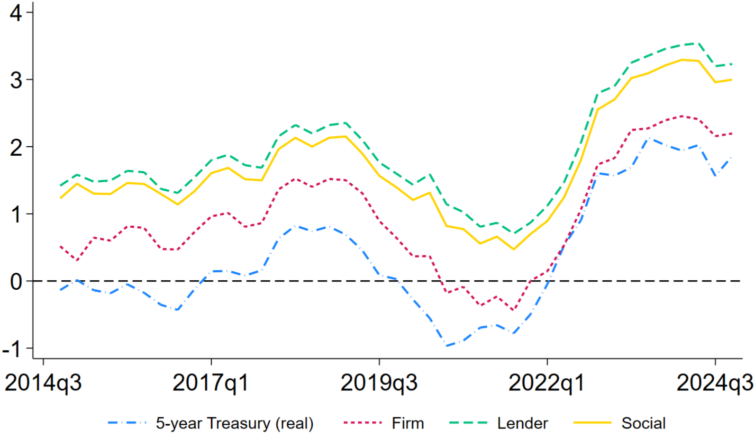


Estimates for lender discount rate, firm and social cost of capital

	Mean	SD	p10	p50	p90
ρ (%)	1.87	1.55	0.41	1.88	3.62
r^{firm} (%)	0.92	2.80	-0.86	1.26	3.03
r^{social} (%)	1.66	1.78	0.12	1.73	3.47

- Financial frictions/recovery: $\mathbb{E}\left[r_{i,t}^{firm}\right] < \mathbb{E}\left[r_{i,t}^{social}\right], \mathbb{E}\left[\rho_{i,t}\right]$

Validation: r^{social} correlates with standard measures of ARPK

▷ details

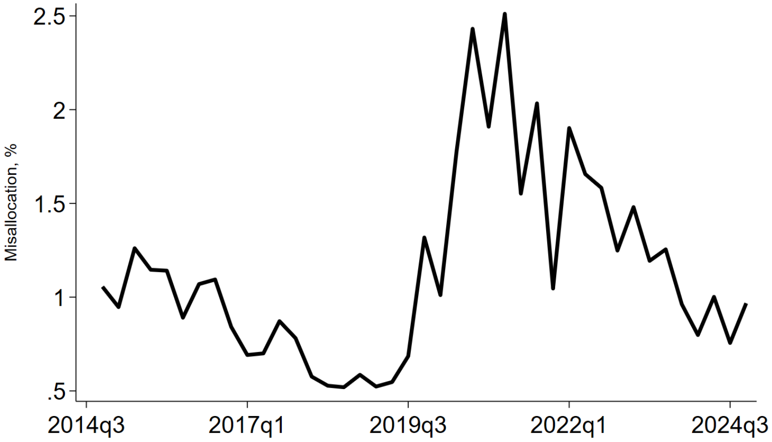
	(1)	(2)	(3)	(4)	(5)
	$\log(ARPK)$, Sales	$\log(ARPK)$, EBITDA	$\log(ARPK)$, Sales	$\log(ARPK)$, EBITDA	$\log(ARPK)$, VA
$\log(r^{social} + \delta)$	0.15***	0.24***	0.16**	0.15*	0.39***
	(0.03)	(0.04)	(0.07)	(0.08)	(0.07)
Observations	59294	57334	4184	4072	3432
Adj. R2	0.27	0.22	0.68	0.52	0.61
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Misallocation in the US, 2014-2024

▷ weighted



- About 0.9% before 2020
- ↑ to 1.8% in 2020-2021
- ↓ to 1.2% in 2022-2024

The 2020–2021 increase in misallocation

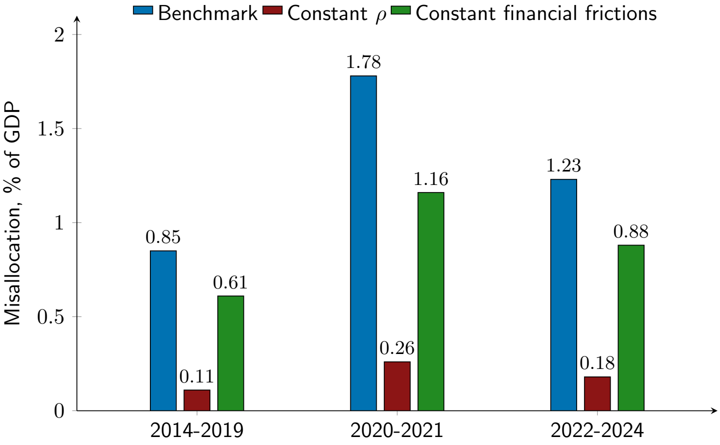
1. Driven by dispersion in lender discount rates ρ_i, not financial frictions.

2. Sharp rise in the coefficient of variation of ρ_i.

3. Variance of ρ_i increases due to increased dispersion of expected losses.

1. The 2020-21 rise in misallocation was driven by $\{\rho_i\}$

▷ details



- Main driver: dispersion in lender discount rates
- Interaction between ρ_i and financial frictions $(0.85 > 0.11 + 0.61)$

2. The CV of ρ_i increased during 2020-21

- Policy rates $\downarrow$ in 2020-21 $\Rightarrow$ mean $\rho_i \downarrow$
- $\sigma(\rho_i) \uparrow$ during this period - why?

$\Rightarrow$ 2. Coefficient of variation of $\rho_i \uparrow$

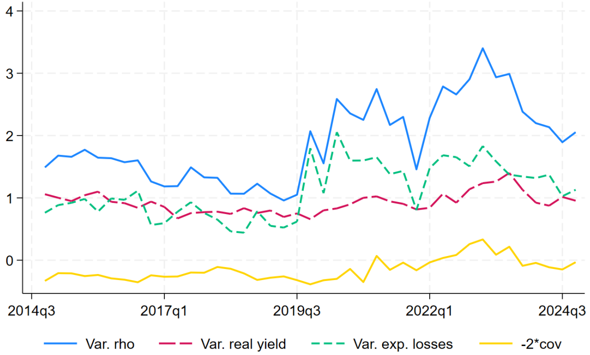
3. Variance of ρ related to variance of expected losses

- Compute "real yield" $\rho^*_{i,t}$: lender discount rate if no default

▷ real yield

- Decomposition: $\rho_i = \underbrace{\rho^*_i}_{\text{real yield}} - \underbrace{[\rho^*_i - \rho_i]}_{\text{exp. losses}}$

Variance decomposition of rho



Variance of ρ_i:

$\mathbb{V}\,[\text{yield}] + \mathbb{V}\,[\text{exp. losses}] - 2\mathbb{C}\,[\text{yield}, \text{exp. losses}]$

- Increase in variance explained by exp. losses
- Covariance falls in absolute value
- $\uparrow$ in dispersion of exp. losses without $\uparrow$ in dispersion of contractual rates

Outline

Extensions & robustness

1. Estimate heterogeneous price-impact term $\mathcal{M}$. ▷ heterogeneous $\mathcal{M}$
2. Variance decomposition: dispersion accounted by bank, firm, loan. ▷ variance decomposition
3. Application to cross-country data. ▷ details

Work in progress:

1. Aggregate risk
2. Quantitative model

Conclusion

- Framework to measure misallocation from credit registry data.

  1. Standard dynamic corporate finance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Uses standard credit registry variables $(r, P, LGD, T, \ldots)$

- Application to U.S. credit registry data

  1. Estimate lender discount rates, firm-level cost of capital and social cost of capital
  2. Misallocation around 1% in normal times
  3. Rise in 2020-21, driven by increase in variance of expected losses

Credit markets in the US appear efficient, but crisis interventions can amplify misallocation.

Appendices

Firm FOC: details

▷ back

Firm FOCs:

$$[k_i']: \quad -1 + \frac{\partial Q_i(k_i', b_i', z_i)}{\partial k_i'}[b_i' - (1-\theta_i)b_i] + \beta \mathbb{E}\left\{ \mathcal{P}_i(k_i', b_i', z_i')[f_k(k_i', z_i') + 1 - \delta] \middle| z_i \right\} = 0$$

$$[b_i']: \quad \frac{\partial Q_i(k_i', b_i', z_i)}{\partial b_i'}[b_i' - (1-\theta_i)b_i] + Q_i(k_i', b_i', z_i) - \beta \mathbb{E}\left\{ \mathcal{P}_i(k_i', b_i', z_i')[\theta_i + (1-\theta_i)Q_i(k_i'', b_i'', z_i')] \middle| z_i \right\}$$

$$= 0$$

Firm's cost of capital: proof

▷ back

$$\frac{1}{Q_t}\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]=\frac{(1+\rho)\,\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]+\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}$$
$$=(1+\rho)\left(1+\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}\right)^{-1}$$
$$=(1+\rho)\,(1+\Lambda)^{-1}$$

where

$$\Lambda\equiv\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}$$

Misallocation formula: proof

▷ back

- Formally, planner's problem is now the same as solving $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i)\mathrm{d}i$, where $f_i(k_i)$ is now expected output

- Apply Hughes and Majerovitz (2024), noting $\frac{dY}{dk} = r^{social} + \delta$

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right] + \delta\right)^2}\right)$$

- $\mathcal{E}$ is (negative) elasticity of output w.r.t. cost of capital $(r^{social} + \delta)$

Calibration of $\mathcal{E}$

▷ back

- $\mathcal{E}_i$ is the elasticity of expected output with respect to the cost of capital

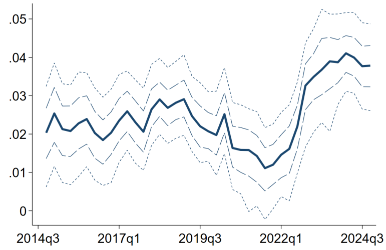
- Assume that $f(k, z) = z \cdot k^{\alpha}$ and there is no default, then

$$\mathcal{E} = \frac{\alpha}{1-\alpha}$$

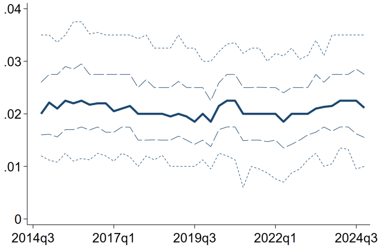
- $\alpha = \frac{1}{3}$ implies $\mathcal{E} = \frac{1}{2}$

Time series for averages and quantiles: real interest rate, PD, LGD ▷ back

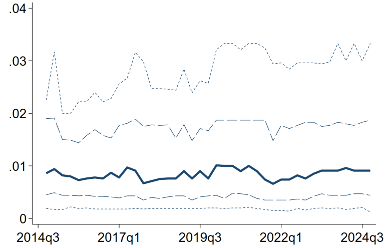
Real interest rate



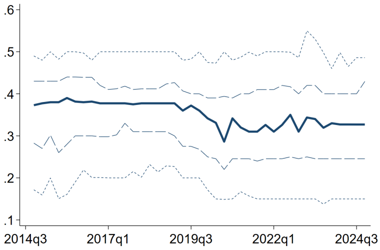
Interest rate spread (var.)



Probability of default



Loss given default



Data cleaning and sample construction

▷ back

We use FR Y-14Q Schedule H.1 data from 2014Q4 to 2024Q4.

Borrower Filters:

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

Data cleaning and sample construction, cont'd

▷ back

Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only "vanilla" term loans (Facility type $= 7$)
- Drop loans with:
  - Mixed-interest rate structures
  - Maturity less than 1 year or longer than 10 years
  - Implausible interest rates or spreads (outside 1st - 99th percentile)
  - Missing or invalid PD/LGD values (outside $[0, 1]$)
  - PD $= 1$ (flagged as in default)

Forward interest rate expectations

▷ back

To estimate ρ_i for floating rate loans, need estimates of $\mathbb{E}_0\left[r_t\right] + s_i$

- Floating rate loans charge reference rate + spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak, Sack, and Wright, 2007)
- Average spread between SOFR and Treasury rates 2018-2025 $\simeq$ 2 basis points
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out $\mathbb{E}_0\left[r_t\right] + s_i$ for each loan, using treasury forward rate plus loan's spread

Firm cost of capital: model

▷ back

$$Q_t = \frac{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta)\, Q_{t+1}\right) + \left(1 - \mathcal{P}_{t+1}\right) \phi k_{t+1}/b_{t+1}\right]}{1+\rho}$$

Note that

$$\begin{aligned}
Q_t &= Q_t^P + Q_t^D \\
Q_t^P &= \frac{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta)\, Q_{t+1}\right)\right]}{1+\rho} \\
Q_t^D &= \frac{\mathbb{E}_t \left[\left(1 - \mathcal{P}_{t+1}\right) \phi k_{t+1}/b_{t+1}\right]}{1+\rho}
\end{aligned}$$

That is, we strip the bond into the payment in repay $\left(Q_t^P\right)$ and the payment in default $\left(Q_t^D\right)$. Then:

$$\Lambda = \frac{\mathbb{E}_t \left[\left(1 - \mathcal{P}_{t+1}\right) \phi k_{t+1}/b_{t+1}\right]}{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta)\, Q_{t+1}\right)\right]} = \frac{Q_t^D}{Q_t^P}$$

Firm cost of capital: measurement

▷ back

The firm defaults with probability $(1-P)$ and the lender recovers $(1-LGD)$. Hence

$$Q_t^{D,data} = \frac{(1-P)(1-LGD)}{1+\rho}$$

For the payment portion notice that at issuance we have the following condition

$$1 = \sum_{s=1}^{T}\left[\frac{P^s\mathbb{E}_t[r_{t+s}] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}$$

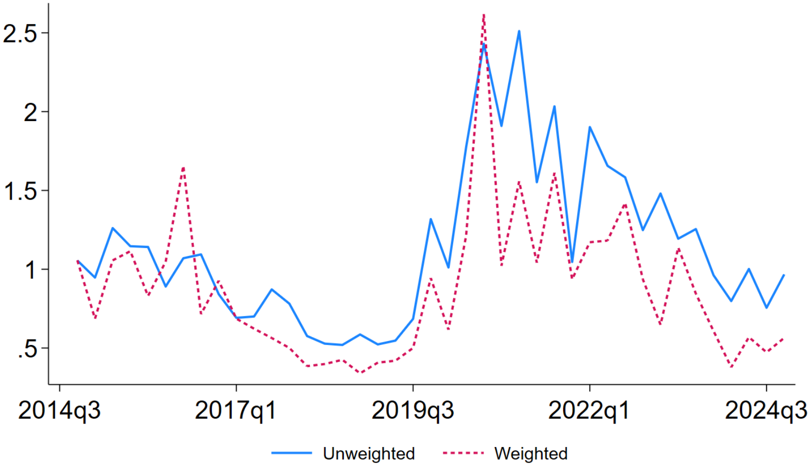
$$1 = \frac{(1-P)(1-LGD)}{1+\rho} + P\frac{\mathbb{E}_t[r_{t+1}]}{1+\rho} + \left(\sum_{s=2}^{T}\left[\frac{P^s\mathbb{E}_t[r_{t+s}] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}\right)$$

So, we can define $Q_t^{P,data}$ as $1 = Q_t^{P,data} + Q_t^{D,data}$ so $Q_t^{P,data} = 1 - Q_t^{D,data}$. Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1-P)(1-LGD)}{1+\rho-(1-P)(1-LGD)}$$

Misallocation, weighted by loan size

▷ back



Decomposing misallocation

▷ back

Counterfactual I: What if all lenders have the same $\bar{\rho}$?

$$1 + r_{social}^{cf,I} = \overline{(1+\rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in r_{social}^{cf} $\rightarrow$ Misallocation due to financial frictions

Counterfactual II: what if we equalize financial frictions?

$$1 + r_{social}^{cf,II} = (1+\rho)\,\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in r_{social}^{cf} $\rightarrow$ Misallocation due to heterogeneous cost of capital

"Real yield": decomposing ρ

▷ back

- The "real yield" is the implied $\rho^*_{i,t}$ when $P_{i,t} = 1$

$$1 = \sum_{s=1}^{T_{i,t}} \left[\frac{\mathbb{E}_t\left(r_{i,t,s}\right)}{\left(1+\rho^*_{i,t}\right)^s \cdot \mathbb{E}_t(\Pi_{t,s})} \right] + \frac{1}{\left(1+\rho^*_{i,t}\right)^{T_{i,t}} \cdot \mathbb{E}_t(\Pi_{t,T_{i,t}})}$$

- Real yield independent of $P_{i,t}, LGD_{i,t}$

- Only affected by losses through the contractual rate r

Estimating $\mathcal{M}$

▷ back

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \times \frac{\partial \log Q}{\partial \log b'}}$$

Need Q, γ, and firm leverage Qb'/k' to compute $\mathcal{M}$

1. To compute Q, assume that loans are perpetuities that decay at a geometric rate θ, discounted at the loan's real interest rate r:

$$Q = \frac{\theta + (1-\theta)Q}{1+r} = \frac{\theta}{r+\theta}$$

   r is directly observed in the data, and we can approximate $\theta = 1/T$

2. Guess a functional approximation $Q(z, k, b, \rho)$
3. Estimate $\log \hat{Q}(z, k, b, \rho)$ for every loan origination; compute partial derivatives
4. At steady state, $\gamma = \theta = 1/T$

Estimating $\mathcal{M}$: Q elasticities

▷ back

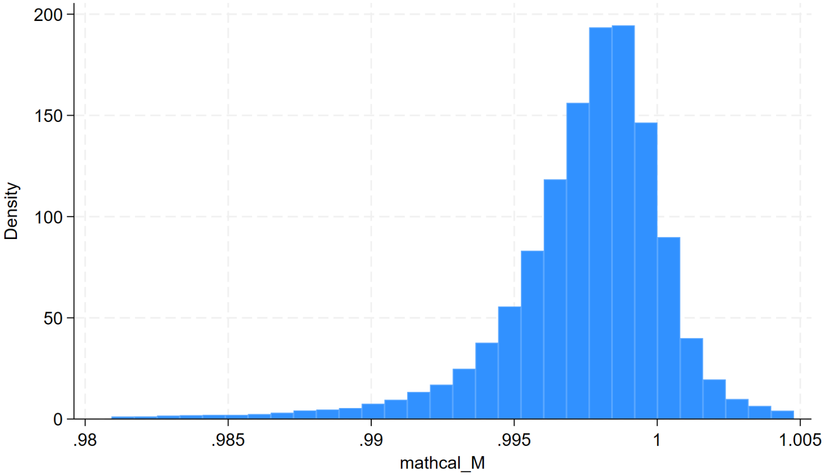
- We approximate (the log of) Q as a polynomial of firm capital, borrowing, productivity and ρ

$$
\begin{aligned}
\log Q_i = \alpha &+ \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\
&+ \beta_{k,k} (\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\
&+ \beta_{b,b} (\log b_i)^2 + \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\
&+ \beta_{z,z} (\log z_i)^2 + \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho} (\rho_i)^2 + \epsilon_i
\end{aligned}
$$

- Capital: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets
- This allows us to compute $\frac{\partial \log Q}{\partial \log k'}$ and $\frac{\partial \log Q}{\partial \log b'}$

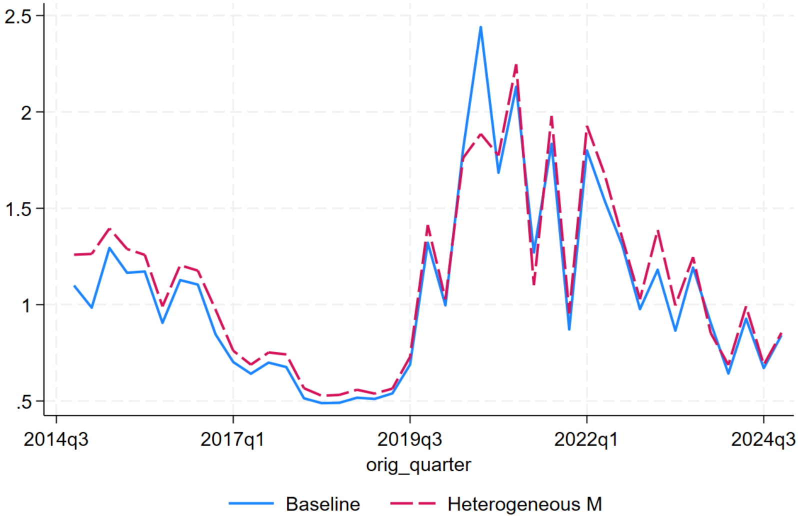
Estimating $\mathcal{M}$: results

▷ back



Misallocation with heterogeneous $\mathcal{M}$

▷ back



Variance decomposition

▷ back

	Time	Bank	Firm	Loan
Contractual rate	69	2	15	15
Real rate	49	4	25	22
ρ	43	4	23	30
r^{firm}	17	4	31	49
r^{social}	35	4	25	36

Notes: 1,844 firms and 16,088 loans. Sample restricted to firms with at least five securities.

Within-period dispersion of r^{social}:

- Bank 6%
- Firm 38%
- Loan 55%

Large dispersion even within a quarter-bank-firm relationship.

ARPK-based misallocation

▷ back

Focus on Compustat firms to make measures comparable

	$r^{social} + \delta$	$\frac{\text{Sales}}{\text{Capital}}$	$\frac{\text{EBITDA}}{\text{Capital}}$	$\frac{\text{Value Added}}{\text{Capital}}$
$Var(\log)$	0.01	0.19	0.24	0.21
Misallocation (%)	0.36	4.75	6.20	5.28

- Our measure looks only at misallocation coming from heterogeneity in the cost of capital
- ...but does not require detailed data on firm financials (i.e., value added)
- $\implies$ directly applicable to most existing credit registries

ARPK-based misallocation measures

▷ back

	(1) log $ARPK$, Sales	(2) log $ARPK$, EBITDA	(3) log $ARPK$, Sales	(4) log $ARPK$, EBITDA	(5) log $ARPK$, VA
$\log(r^{social} + \delta)$	0.15^{***}	0.24^{***}	0.16^{**}	0.15^{*}	0.39^{***}
	(0.03)	(0.04)	(0.07)	(0.08)	(0.07)
Observations	59294	57334	4184	4072	3432
Adj. R2	0.27	0.22	0.68	0.52	0.61
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat
Var($\log ARPK$)	1.97	1.52	0.19	0.24	0.21
Misalloc., $ARPK$, %	63.63	46.08	4.75	6.20	5.28
Var($\log(r^{social} + \delta)$)	0.04	0.04	0.01	0.01	0.01
Misalloc., $r^{social} + \delta$, %	0.96	0.96	0.36	0.36	0.36

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Cross-country comparison, approximation

▷ details ▷ back

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
Mean real rate, %	66.8	8.00	83.0	12.4	1.4
SD real rate, %	38.1	2.9	93.3	5.2	1.2
Mean def. prob., %	2.7	16.9	4.0	8.9	1.5
Mean recovery rate, %	42.8	42.8	18.2	63.9	66.6
Implied misallocation, %	6.5	13.5	21.5	2.8	0.8

- Developing countries: higher mean and standard deviation of real interest rates
- U.S.: lower mean and standard deviation of interest rates, higher recovery
- Brazil: most extreme misallocation: 21.5%.

Details on cross-country comparison

▷ back to loan pricing ▷ back to E&R

- For a fixed real interest rate $r_{i,t}$, ρ has a closed-form:

$$1 + \rho_{i,t} = P_{i,t}\left(1 + r_{i,t}\right) + \left(1 - P_{i,t}\right)\left(1 - LGD_{i,t}\right)$$

- Assume all loans have the same maturity:
  1. Obtain mean real rate by subtracting average realized inflation from mean nominal rate
  2. Inflation should not affect standard deviation of nominal rates (or spreads)
- Assume all loans have the same $P_{i,t}, LGD_{i,t}$, equal to the average
- Recovery rates and inflation rates from the World Bank
- Approximate $r_{i,t}^{social} \simeq \rho_{i,t}$ and compute misallocation using our formula:

$$\log(Y_t^*/Y_t^{DE}) = \frac{1}{2}\mathcal{E}\log\left(1 + \frac{Var(\rho_{i,t})}{(\mathbb{E}[\rho_{i,t}] + \delta)^2}\right)$$